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Good Enough Jobs: Skill Mismatch and Two-Sided  
Heterogeneity in Frictional Labor Markets

by

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# Good Enough Jobs: Skill Mismatch and Two-Sided Heterogeneity in Frictional Labor Markets

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## **Abstract**

In this paper, I explore the existence and prevalence of skill mismatch in the labor market both empirically and theoretically. Using data from the NLSY97 and O\*NET, I show that while there is some degree of positive sorting, matching is far from perfect. Higher-skilled workers tend to be employed in better jobs, but many accept poor-quality matches in order to exit unemployment more quickly. This leads to lower unemployment rates and increased wage dispersion among higher-skilled workers. In light of these stylized facts, I construct a labor search model to formalize the workers' tradeoff between accepting an imperfect match and continuing to search. The model is solved numerically and calibrated to U.S. labor market. The calibrated model generates match acceptance strategies and employment outcomes that are broadly consistent with the empirical facts.

*PRELIMINARY AND INCOMPLETE*

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# 1 Introduction

At most a third of the variation in wages across workers is explained by the observable characteristics of the worker<sup>1</sup>. Skill mismatch, measured by the difference between the skills of the worker and the skill requirements of the job in which he is employed, indicates a lower match quality. Recent empirical evidence has suggested that skill mismatch can account for part of the remaining wage dispersion. However, the prevalence of skill mismatch in the labor market has not yet been quantified.

This paper presents three stylized facts describing skill mismatch in the U.S. labor market, and constructs a search and matching model to replicate these facts. I use data from the 1997 cohort of the National Longitudinal Survey of Youth (NLSY97) to determine empirically how much skill mismatch workers are willing to accept. I find that mismatch is present among workers of all skill levels, and that higher-skilled workers tolerate more skill mismatch than lower-skilled workers. Workers' average wages decline as mismatch increases; therefore, within-type wage dispersion is higher for higher-skilled workers. In addition, higher-skilled workers experience shorter unemployment spells, and are less likely to be unemployed at any time.

In light of these stylized facts, I construct a labor search model that formalizes the tradeoffs faced by workers and firms between accepting an imperfect match and continuing to search. The model builds off of the canonical search-and-matching framework of McCall (1970), augmented to allow for heterogeneity in the skills of both workers and firms. The model focuses on the worker's problem, specifically on that of the unemployed worker. Unemployed workers search randomly for jobs, meeting vacant jobs according to a Poisson arrival rate. Upon meeting for an interview, an unemployed worker chooses whether to begin employment with the current firm or to continue searching. In this environment, it is not an equilibrium strategy for an agent to wait for her "perfect" match. Instead, workers will choose to accept some range of "good enough" matches.

I solve the model analytically and calibrate it to the post-recession U.S. labor market using NLSY97 data from 2009 to 2013. The calibrated model generates match acceptance sets that are broadly consistent with the data. In addition, the expected level of mismatch for each worker type is similar to the average mismatch in the data for all but the highest-skilled workers. In the case of these workers, the model-generated mismatch is approximately 50% higher than mismatch observed in the data; on average, high-skilled workers are able to sort into good jobs more effectively than the model predicts. This discrepancy suggests a potential

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<sup>1</sup>References!

improvement upon the assumed search process, which will be explored in subsequent work. In line with the stylized facts, the model also generates unemployment rates and durations that are decreasing in the worker’s skill type.

This simple model provides an ideal framework to explore a variety of questions regarding the role of skill mismatch, such as the relationship between unemployment insurance policy, skill mismatch, and aggregate productivity. I use the calibrated model to perform a counterfactual analysis of the effect of unemployment insurance (UI) benefits on mismatch and wages. Workers respond in an intuitive way to changes in UI policy; lower UI benefits encourage workers to be less selective in accepting job offers in order to exit unemployment more quickly, and increased benefits subsidize search and make workers more choosy. The model can accommodate variation in UI benefits across workers, making possible the investigation of a wide range of UI policy structures. For example, in the calibrated model I set UI benefits equal to 40% of the worker’s maximum wage. It is also possible selectively increase (or decrease) the replacement rate for certain ranges of worker types. Specifically, I consider an example where the replacement rate increases linearly in worker type; in this case, the unemployment rates of lower-skilled workers fall, while higher-skilled workers become more selective. These results suggest an important role for skill mismatch in the determination of optimal UI policy.

The remainder of this paper is organized into the following sections. Section 2 begins by constructing three empirical facts about the presence of skill mismatch in the U.S. labor market. Section 3 presents a model of one-sided search with heterogeneity and illustrates the decentralized equilibrium solution in a calibrated numerical example. Section 4 provides a counterfactual analysis of the impact of unemployment insurance benefits on aggregate mismatch through the role of a search subsidy, and Section 5 concludes.

## 1.1 Related Literature

Albrecht and Vroman (2002) introduce a simple model of job search by two types of vertically differentiated workers, successfully generating skill mismatch as well as wage dispersion among the high-skilled workers. In an extension with on the job search, Gautier (2002) examines welfare implications of the “crowding-out” effect of high-skill workers in the unskilled labor market.

The random search model developed by Marimon and Zilibotti (1999) allows worker and firm types to differ continuously along a circle. The primary drawback of this model is the inability to address qualitative differences between different types of workers or jobs.

A number of subsequent skill mismatch papers build off of this approach. Under directed search, Decreuse (2008) shows that unemployed workers accept too much mismatch relative to the social planner’s allocation due to a composition externality. To determine the impact of mismatch on aggregate productivity, Gautier and Teulings (2015) augment the circular model to allow for on the job search and calibrate it using U.S. labor market data.

Search models with a hierarchy of types have been studied in the marriage search literature, beginning with Shimer and Smith (2000) and Shimer and Smith (2001). The authors prove existence of an equilibrium, derive sufficient conditions for positive assortative matching, and show that the decentralized equilibrium is never efficient. Teulings and Gautier (2004) extend the model to a labor market setting and specify an increasing returns to scale matching process in order to approximate the decentralized equilibrium solution and estimate the distortions resulting from search frictions.

Eeckhout and Kircher (2011) examine sorting in the context of a labor search market with skill heterogeneity, and conclude that it is not possible to determine from wage data alone whether the equilibrium features positive or negative sorting. Using a model with exogenous productivity shocks and on-the-job search, Lise, Meghir, and Robin (2016) exploit variation in workers’ wages and job transitions to argue that sorting is present in the market for skilled workers (those with at least a high school degree) but not present for unskilled workers. This suggests that complementarity between worker and firm skills is important only for skilled workers.

In a model of occupational choice where workers learn about their abilities in verbal, math, and social skills over time, Guvenen, Kuruscu, Tanaka, and Wiczer (2015) conclude that mismatch in each of these dimensions has a negative effect on a worker’s current wage and lifetime earnings. Lise and Postel-Vinay (2015) employ a search model with multidimensional skills to examine the wage penalty for skill mismatch in cognitive, manual, and interpersonal skills. The authors find that mismatch along the cognitive skill dimension is more costly and slower to correct through skill accumulation, compared to mismatch in manual skills.

## **2 Skill Mismatch in the U.S. Labor Market**

In this section, I construct empirical measures of worker and firm skills, and summarize stylized facts regarding skill mismatch in the labor market. I use data from the NLSY97 as well as the U.S. Department of Labor’s O\*NET project.

## 2.1 Data

The primary data source in this paper is the NLSY97<sup>2</sup>. Conducted by the Bureau of Labor Statistics, this nationally representative survey samples individuals born between 1980-1984. To ensure that most of the individuals in the sample have entered the labor market, I restrict the analysis to the 2009-2013 waves of the survey. During this time period, respondents are 25-33 years old.<sup>3</sup> Table 4 in Appendix A provides descriptive statistics for the sample.

To create a measure of skill that is comparable across individuals, I follow an approach similar to that of Cawley, Heckman, and Vytlačil (2001). During the first round of the survey, most respondents took the Armed Services Vocational Aptitude Battery (ASVAB) test. The ASVAB consists of 12 component sections, over skills both abstract (eg. mathematics knowledge, paragraph comprehension) and practical (eg. auto information). Scores from four categories, mathematics knowledge, arithmetic reasoning, paragraph comprehension, and word knowledge, are residualized by age and gender<sup>4</sup>, and a principal components analysis is performed on the residuals. The individual’s percentile rank in the first principal component score is referred to as the ASVAB rank.

I obtain information on occupational skill requirements from O\*NET. Using a random sample of workers within each occupation, O\*NET provides information on 704 SOC-level occupations, 462 of which are represented in the NLSY97 subsample used here<sup>5</sup>. For each occupation, a “level” and an “importance” score are provided for each of 277 descriptors. The level score assigned to a skill indicates the degree of competency in that skill needed to succeed at the occupation; the importance score describes how essential the skill is to the occupation. For example, the skill “Mathematics” is rated as equally important for both Physicists and Post-secondary Mathematics Teachers, but the level of the skill required is substantially higher for Physicists. I will focus on the level requirement for the skill(s) of interest.

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<sup>2</sup>With the exception of Lindenlaub (forthcoming), most empirical studies to date on skill mismatch in the labor market have used the NLSY79 for analysis. Respondents in the NLSY97 have now entered their prime working years, which makes the data appropriate for studying labor market outcomes.

<sup>3</sup>There is some attrition in the sample; out of the initial 6,748 individuals in the cross-sectional sample, approximately 6,000 are successfully contacted in 2009. To account for selective attrition, I use custom sample weights calculated over the subsample of individuals who appear in any wave between 2009-2013. Most individuals in this subsample are active in the labor market; in 2009, 5,901 respondents reported receiving some type of labor income.

<sup>4</sup>See Appendix B for a discussion of alternatives.

<sup>5</sup>Occupational codes in the NLSY97 are reported as 3- or 4-digit Census codes, a coarser taxonomy than used by the SOC. For occupations that map to multiple SOC codes, I use the average of skill requirements.

## 2.2 Empirical Methodology

Skill mismatch is characterized as the difference between a worker’s skill type and the skill type of the occupation in which the worker is employed. For simplicity, both skill types are represented by linear indices,  $x$  and  $y$  respectively. Skill mismatch is defined as  $x - y$ ; a positive value indicates that the worker is over-skilled relative to his job, and a negative value indicates that the worker is under-skilled. In order to measure and describe skill mismatch in the labor market, measures of worker and job skill types must first be constructed.

I interpret worker skill as the amount of cognitive human capital the worker possesses. ASVAB rank provides some information about a worker’s skill type. However, since the ASVAB was administered before most educational attainment decisions were made, it is likely that rankings shifted prior to 2009. To account for this, I combine the ASVAB rank with the respondent’s education level using principal component analysis, where the first component is taken as the individual’s general ability. Finally, I recompute the ranking of individuals using the custom sample weights previously described, and normalize to obtain a skill type  $x \in [0, 1]$ . This method returns a ranking over individuals such that, conditional on education level, an individual with a higher ASVAB rank is ranked more highly in general ability<sup>6</sup>. Figure 1 shows the distribution of educational attainment within each percentile

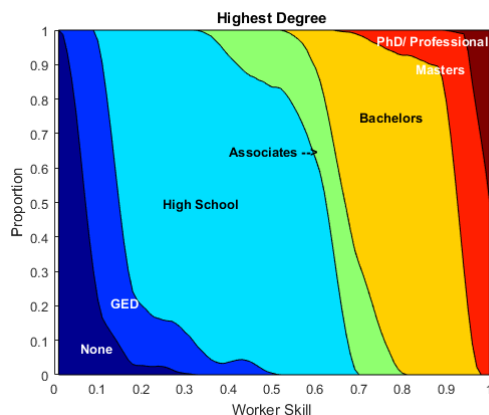


Figure 1: Distribution of educational attainment by worker skill type.

bin of worker skills, smoothed using nonparametric local-constant least squares regressions. Workers classified as higher-skilled tend to have higher levels of educational attainment, but the ordering is not perfect. It is possible for a worker with a high school degree and a relatively high ASVAB rank to be classified near the top of the skill space, above workers with college degrees but low ASVAB ranks.

<sup>6</sup>See Appendix A for a discussion of other ranking methods.

A job equivalent to an occupation, or a group of tasks that the worker must perform. For the purpose of this analysis, the skill type of a job is determined by the levels of the skills “Judgment and Decision Making” (JDM) and “Cognitive Problem Solving” (CPS) required by that occupation<sup>7</sup>. These values are combined using principal components, and the first principal component is taken to be the intellectual skill requirement of the occupation. JDM is the skill most strongly (positively) correlated with the occupation’s median hourly wage; CPS is highly correlated with both JDM and the wage, and is included primarily to increase variation in occupation skills. Occupations are weighted according to employment share (reported by the Bureau of Labor Statistics) and a ranking is computed; after normalization, job skill types  $y$  also span the interval  $[0,1]$ . Skill mismatch can then be calculated as the difference between the worker’s skill  $x$  and the skill  $y$  of the occupation in which the worker is currently employed.

The NLSY records respondents’ hourly compensation at each job, including wages, overtime, tips, and bonuses. The variable is calculated by the NLSY using the respondent’s time unit of pay, hours worked per time unit, and usual wages for that unit of time. I drop observations with reported hourly compensation less than \$1 per hour or greater than \$100 per hour as well as jobs where the respondent worked less than 5 hours per week. I adjust for inflation by indexing all values to 2009 dollars. Finally, data on job tenure and unemployment duration are obtained from the NLSY weekly employment arrays.

## 2.3 Stylized Facts

Using the skill measures constructed in the previous section, I document three stylized facts regarding workers’ match acceptance behavior and the presence of skill mismatch in the labor market.

**Fact 1.** Higher-skilled workers are less likely to be unemployed, experience shorter unemployment spells, and earn higher incomes.

Figure 2 plots the average unemployment rate and unemployment duration by worker skill type, and the average of observed hourly compensation for each percentile bin of worker skill type and each percentile bin of occupation skill type. Fact 1 is consistent with previous empirical findings<sup>8</sup>, and helps to establish that the measures of worker and occupation

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<sup>7</sup>See Appendix A for a discussion of rankings using other descriptors and robustness checks.

<sup>8</sup>For example, see Becker (1993) for a discussion of unemployment rates across education levels, or Heckman, Stixrud, and Urzua (2006) for the relationship between unemployment rate and cognitive ability. Cawley et al. (2001) review recent evidence of the effects of cognitive ability on wages.



skill types used here capture meaningful variation in worker abilities and firm requirements, respectively.

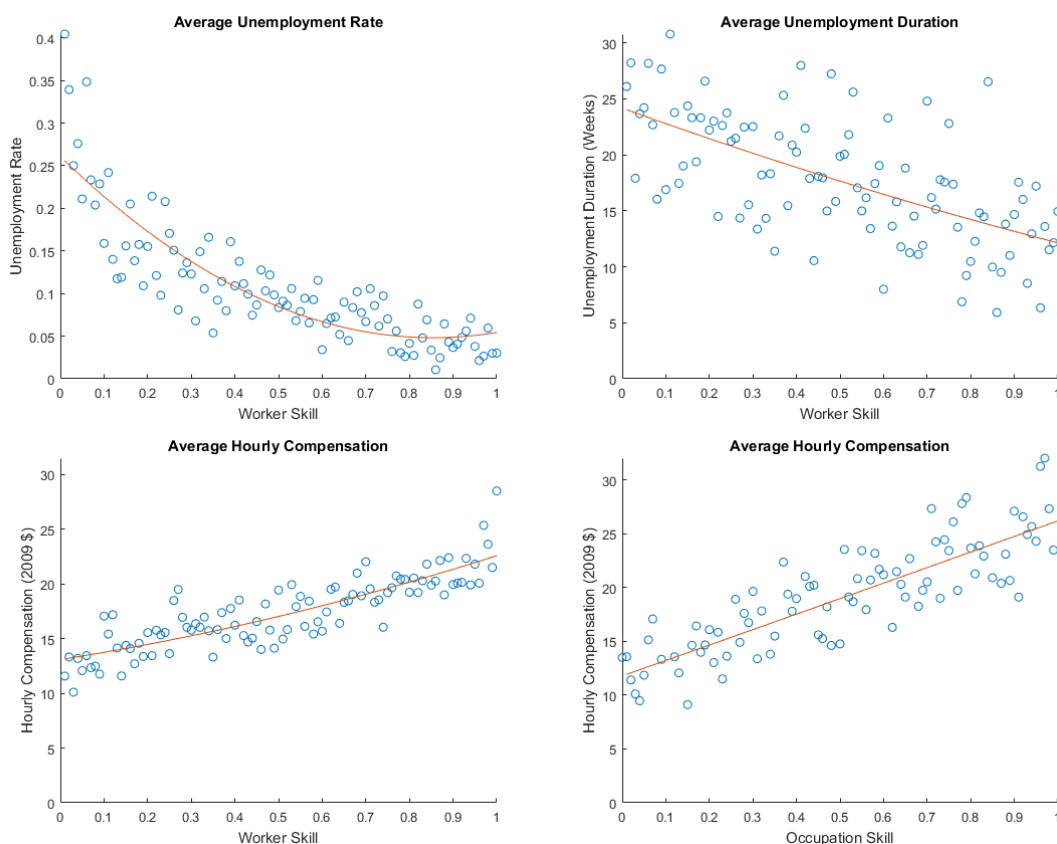


Figure 2: Labor market outcomes across skill types.

**Fact 2.** The 90-10 and 90-50 wage differentials are positively correlated with the worker’s skill type.

The average wage is increasing in the worker’s skill type, but workers face substantial within-type wage differentials. The lowest-skilled workers’ wages range from around \$8 per hour at the 10<sup>th</sup> percentile to about \$22 at the 90<sup>th</sup>; among the highest-skilled workers, the 10<sup>th</sup> and 90<sup>th</sup> percentile individuals earn approximately \$10 and \$35 per hour, respectively. Figure 3 plots the wage differentials between the 90<sup>th</sup> - 50<sup>th</sup> and 50<sup>th</sup> - 10<sup>th</sup> percentiles of hourly compensation within each worker skill type. Both measures of wage dispersion increase almost linearly in skill type; for each decile increase in worker skills, the 90-10 wage differential increases by \$1.07 and the 90-50 wage differential increases by \$0.51. The increasing wage differential suggests that higher-skilled workers are employed in a wider range of jobs.

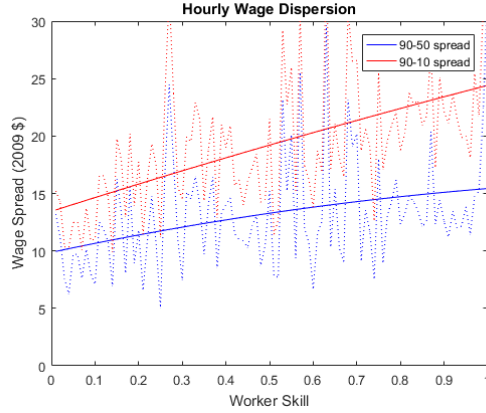
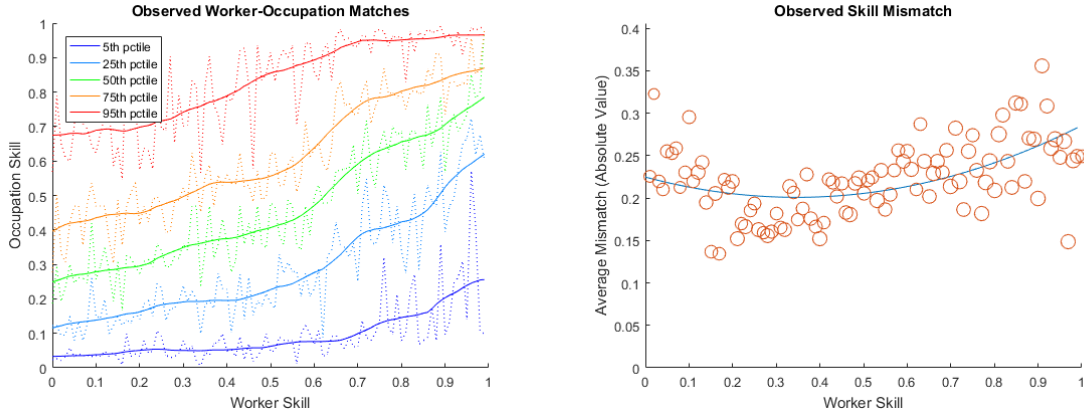


Figure 3: Wage dispersion within worker skill types.

Figure 4: Observed matches in the NLSY97.



(a) Matches between worker and occupation skill types. (b) Average mismatch within each percentile of worker type.

**Fact 3.** There is some degree of positive sorting, but matching is not perfectly assortitative. On average, there is more skill mismatch among higher-skilled workers.

Figure 4a depicts the range of occupation types that each worker type is observed to match with. The figure plots the 95<sup>th</sup>, 75<sup>th</sup>, 50<sup>th</sup>, 25<sup>th</sup>, and 5<sup>th</sup> percentiles of observed occupation skills for each percentile bin of worker skill types (the average bin contains approximately 200 worker-occupation observations). It is clear that some level of positive sorting is present. For each 1% increase in worker skill, the skill level of the median occupation match increases by 0.56%, indicating that higher-skilled workers occupy higher-skilled sets of occupations on average. However, the range of occupations held by a particular type of worker can be quite large; on average, the difference between a worker's 95<sup>th</sup> and 5<sup>th</sup> percentile match is 72.3%

of the occupation skill space. The principal goal of the next section will be to construct a search model that is capable of producing this result.

Figure 4b plots the average of observed skill mismatch within each percentile of worker type. Workers in the lower half of the skill space experience less skill mismatch than higher-skilled workers. The expected level of mismatch for a worker at the 90<sup>th</sup> percentile is 21.8% higher than a worker at the 50<sup>th</sup> percentile, and 16.5% higher than a worker at the 10<sup>th</sup> percentile. This result is surprising, since higher-skilled workers presumably have a higher opportunity cost of mismatch. However, the opportunity cost of unemployment is also higher for high-skilled workers. Unemployment insurance policies attempt to alleviate this by subsidizing job search, but many high-skilled workers are still willing to accept a poor match in order to exit unemployment more quickly.

In the remainder of this paper, I construct a search model to explain workers' mismatch acceptance behaviors consistent with the stylized facts described in this section.

### 3 Job Search with Skill Mismatch

I consider a continuous-time, infinite-horizon model with a labor force of measure 1, and augment the one-sided search environment of McCall (1970) to allow for heterogeneity on both sides of the labor market. Workers are risk-neutral and infinitely-lived, and maximize expected discounted utility. A worker can be either employed or unemployed at any time; all unemployed workers search for jobs, and there is no on-the-job search. In this environment, a firm corresponds to one vacant job<sup>9</sup>.

Workers are heterogeneous in skills, indexed by type  $x \in [0, 1]$ , such that higher  $x$  indicates a more skilled worker<sup>10</sup>. Vacant firms are also heterogeneous, indexed by skill type  $y \in [0, 1]$ . The proportion of type  $x_0$  workers who are currently unemployed is given by the unemployment rate  $u(x_0)$ . Similarly, the vacancy rate for firms of type  $y_0$  is defined relative to the labor force. The vacancy rate  $v(y_0)$  is equal to the ratio of type  $y_0$  vacancies to the total number of workers with type  $x = y_0$ . Worker skill types  $x$  and vacancy skill types  $y$  are distributed according to cumulative distribution functions  $L$  and  $G$ , respectively. The distributions of unemployed and employed workers depend on the workers' decision problem, and are not necessarily equal to  $L$ ; these distributions will be characterized as  $F$

<sup>9</sup>Filled jobs are ignored since this environment does not include a firm's decision.

<sup>10</sup>The model could be extended to allow for a vector of skill traits for both the worker and the firm without changing the intuition; see Guvenen et al. (2015) or Lise and Postel-Vinay (2015) for an example of how multidimensional mismatch measures can be derived from O\*NET data.

and  $\tilde{F}$ , respectively. The firm distribution  $G$  includes only vacant firms, and the distribution of vacancies is assumed to be constant over time since the firms' decision is excluded from this environment.

Once filled, all jobs produce the same numeraire output good. However, the quantity of output produced in a period varies based on the skill types of both the worker and the firm. Let the quantity of output produced by worker  $x$  when employed by firm  $y$  be given by  $\rho(x, y)$ ; this is the match productivity. Define  $\mu = |x - y|$  as the skill mismatch of a worker-firm pair. The production function must be continuously differentiable in  $x$  and in  $y$ , and bounded below at 0<sup>11</sup>. For the numerical exercise in Section 3.2, I use

$$\rho(x, y) = \max\{x - \delta\mu^2, 0\} = \max\{x - \delta(x - y)^2, 0\} \quad (3.1)$$

as the production function, where  $\delta$  is a scalar representing the substitutability of skills. Increasing  $\delta$  amplifies the penalty for mismatch, decreasing the range of jobs with which a worker can profitably match. Higher-skilled workers have the ability to be more productive, but the productivity of a worker-firm match depends also on the quality of the match. Match productivity is decreasing in the level of skill mismatch, so there exist some (low-skilled) jobs in which higher-skilled workers are less productive than low-skilled workers. Under this production function, the “perfect” job for a worker of type  $x$  is  $y = x$ . There is no universally “best” job; each worker's skills are best used in a different type of job. Similarly, for each vacancy type  $y$ , there is a single worker type that maximizes the productivity of this job, but there is no universally “best” type of worker.

Agents derive utility only from consumption of the numeraire good, so the wage fully summarizes the attractiveness of any particular match. Wages for each worker-firm pair are proportional to the match-specific productivity, and are distributed over the interval  $[\underline{w}, \bar{w}]$ . Therefore, a worker's wage is increasing in the worker's type and decreasing in the match-specific level of mismatch<sup>12</sup>. This allows a focus on the worker's job acceptance decision while abstracting from wage determination.

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<sup>11</sup>Although including an explicit productivity deduction from mismatch is intuitive, it is a strong assumption in the sense that it substantially changes a worker's preferred match. However, the function that I choose is very similar to others in the mismatch literature. Marimon and Zilibotti (1999) assume a universal base productivity, with an additive deduction for mismatch. Teulings and Gautier (2004) use the same function, substituting worker type for the base productivity.

<sup>12</sup>Eeckhout and Kircher (2011) show that any bargained wage that is monotonic in match surplus will be concave in firm type. Although this section abstracts from the bargaining problem, I use a production function that is consistent with concave wages.

The distribution  $G(y)$  can alternatively be viewed as a distribution of potential mismatch conditional on the worker's type. The shape of the distribution does not change with this redefinition. Similarly, the distribution of potential wages conditional on the worker's type is also known; define this cumulative distribution function as  $\tilde{G}(w|x)$ , with corresponding pdf  $\tilde{g}(w|x)$ .

All unemployed workers receive job offers according to a Poisson rate  $\lambda$ . Workers cannot determine a potential employer's type prior to receiving an offer, so offers are randomly drawn from the worker's conditional wage distribution  $\tilde{G}(w|x)$ .<sup>13</sup> When an offer arrives, the worker must choose to accept or reject the job. As with standard job search models, this decision is characterized by a reservation wage.

Unemployed workers receive benefits  $b(x)$  during each period of unemployment. The amount of benefits received may depend on the worker's skill level; for example, the benefits could be set to  $b(x) = \bar{b} + b \cdot x$ , where  $b$  is a constant replacement rate. This specification allows the benefits to adjust to better match the workers' expected wages, while keeping the worker's value of unemployment independent of previous employment history<sup>14</sup>. The numerical example in section 3.2 uses this benefit structure.

Finally, jobs are terminated according to a Poisson process with arrival rate  $s$ , which is constant across all worker types. When a job is terminated, the worker becomes unemployed and must search for a new job. I set the workers' discount rate equal to  $r$ . See Figure 5 for a diagram of the timing of events.

### 3.1 Equilibrium

In this section I solve for the reservation wage, focusing on a steady-state equilibrium. Define the reservation wage  $w_x^*$  as the lowest wage a worker of type  $x$  will accept. The equilibrium strategy for a worker of type  $x$  is to accept all job offers with wages greater than or equal to  $w_x^*$ , and to reject all offers lower than  $w_x^*$  in order to continue searching for a higher offer. The reservation wage will change with  $x$ , and the set of reservation wages  $\{w_i^*\}_{i \in [0,1]}$  is sufficient to define workers' equilibrium strategies.

I begin with the value of employment for a worker of type  $x$  in a job paying wage  $w$ ,

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<sup>13</sup>Partially-directed search may be explored in an extension to this model. For examples of directed search models, see Moscarini (2001) and Decreuse (2008). Moscarini constructs a model of directed search with two firm types and a continuum of worker types, and Decreuse solves for the equilibrium of a type-circle model under directed search.

<sup>14</sup>Lise et al. (2016) also assume benefits are a function of only the worker's type; Marimon and Zilibotti (1999) and Teulings and Gautier (2004) set equal benefits for all worker types.

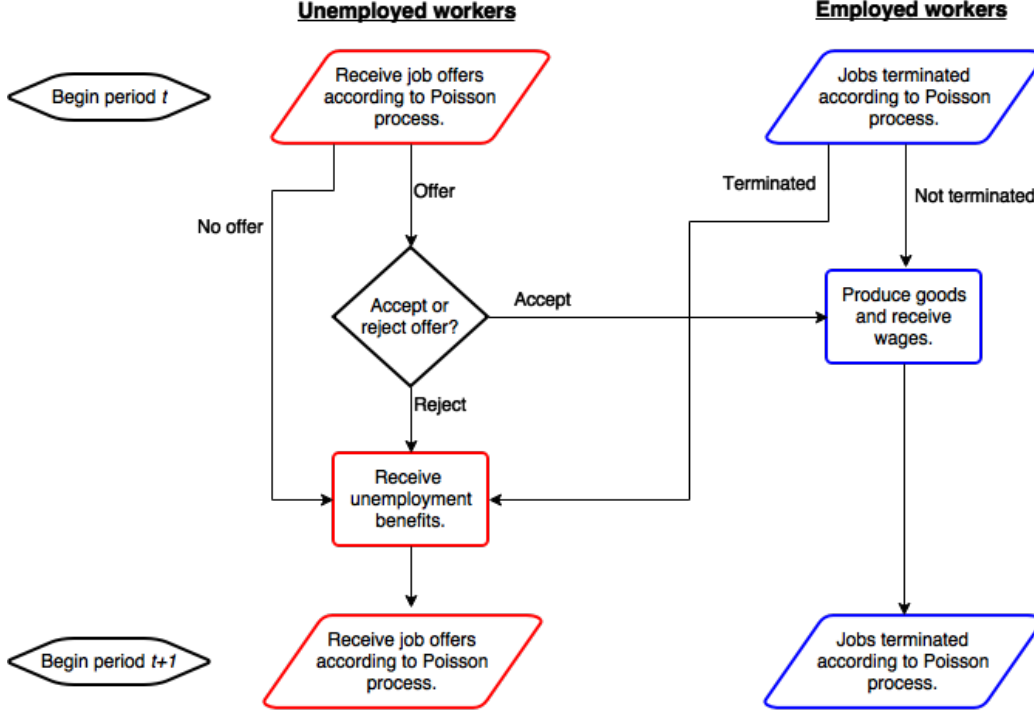


Figure 5: Timeline of events.

$E(x, w)$ . Note that this can be thought of as the value for worker  $x$  employed by firm  $y$ ,  $E(x, w(x, y))$ . The employed worker receives  $w$  in the current period; in the next period, the worker receives either  $U(x)$  or  $E(x, w)$ , with probabilities  $s$  and  $1 - s$  respectively.

$$E(x, w) = \frac{1}{1 + r} [w + s \cdot U(x) + (1 - s) \cdot E(x, w)]$$

In the above equation,  $U(x)$  is the value of unemployment for a type  $x$  worker. Rearranging, the employment value is:

$$E(x, w) = \frac{w + s \cdot U(x)}{r + s} \quad (3.2)$$

Next, I construct the value function for an unemployed worker. The unemployed worker receives unemployment benefits  $b(x)$  in the current period; the continuation value depends on whether an acceptable job offer arrives. An offer  $w$  arrives with probability  $\lambda$ , and is accepted if and only if the value of employment  $E(x, w)$  is greater than the value of remaining unemployed. The ex-ante value of an offer is an expectation with respect to the conditional distribution of wages. If no offer arrives, the continuation value is simply the value of unemployment.

$$U(x) = \frac{1}{1 + r} \left[ b(x) + \lambda \int_{\underline{w}}^{\bar{w}} \max\{U(x), E(x, w)\} d\tilde{G}(w|x) + (1 - \lambda)U(x) \right]$$

This can be solved for the flow value of unemployment:

$$rU(x) = b(x) + \lambda \int_{\underline{w}}^{\bar{w}} \max\{E(x, w) - U(x), 0\} d\tilde{G}(w|x) \quad (3.3)$$

## Reservation Wages

The reservation wage is the minimum wage offer that will induce a worker to accept a job. No two types of workers face the same distribution of possible wages  $\tilde{G}(w|x)$ , so the value of unemployment is a function of the worker's type. It is in a worker's interest to accept all jobs such that the value of employment in that job is at least as great as the worker's value of unemployment, or  $E(x, w) \geq U(x)$ . The lowest wage a worker will accept is the one that sets the value of employment exactly equal to the value of unemployment.

$$\frac{w + sU(x)}{r + s} = U(x)$$

This implies that the reservation wage is equal to the flow value of unemployment, or  $w_x^* = rU(x)$ . Because the  $rU(x)$  is unique for each type of worker, the reservation wage is also unique. Returning to equation 3.3,

$$rU(x) = w_x^* = b(x) + \lambda \int_{\underline{w}}^{\bar{w}} \max\{E(x, w) - U(x), 0\} d\tilde{G}(w|x)$$

After some rearranging,

$$w_x^* = b(x) + \frac{\lambda}{r + s} \left[ \int_{w_x^*}^{\bar{w}} 1 - \tilde{G}(w|x) dw \right] \quad (3.4)$$

The above equation implicitly defines the reservation wage  $w_x^*$  as a function of  $b(x)$ ,  $\lambda$ ,  $r$ , and  $s$ , given the conditional distribution of wages  $\tilde{G}$ .

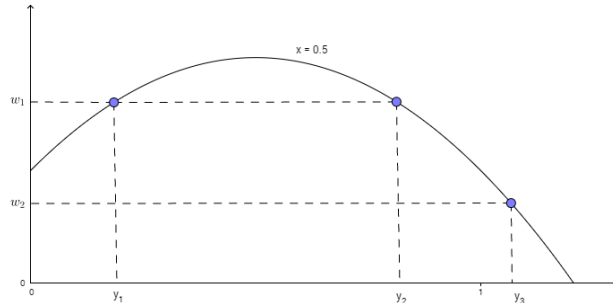


Figure 6: Mapping wage offers back to firm types.

By choosing to accept a reservation wage, the worker implicitly chooses ‘reservation types’ of firms to accept. For a particular worker  $x$ , a wage  $w$  can be obtained from (at most)

two firms. Figure 6 illustrates wages mapped to multiple firm types for worker type  $x = 0.5$  under the productivity function given by equation 3.1. Holding the worker's type constant, wages are assumed to be concave in firm type, consistent with Eeckhout and Kircher (2011).

Clearly,  $w_1$  can be obtained by firms  $y_1$  and  $y_2$ . Although the line through  $w_2$  intersects the wage function,  $y_3$  is outside the assumed range of firm types, so  $w_2$  is outside of the wage space for this type of worker. Of particular interest here are the firms which would offer the worker a wage equal to his reservation wage. Define  $\underline{y}_x$  and  $\bar{y}_x$  as the lower and higher of the 'reservation firms', respectively. Define  $\mathbf{y}$  as the set of firms such that for each firm type  $y^* \in \mathbf{y}$ ,  $w(x, y^*) = w_x^*$ . Then the 'reservation firms' can be expressed as

$$\underline{y}_x = \max(\min(\mathbf{y}), 0) \quad \text{and} \quad \bar{y}_x = \min(\max(\mathbf{y}), 1).$$

Because wages are monotonically decreasing in the level of mismatch, a worker will accept offers from all firms in the range  $[\underline{y}_x, \bar{y}_x]$ . In general, this problem is not symmetric; when the reservation wage would be offered from a firm outside  $[0, 1]$ , it will be the case that  $|x - \underline{y}_x| \neq |x - \bar{y}_x|$  and therefore  $w(x, \underline{y}_x) \neq w(x, \bar{y}_x)$ .

Finally, the expected wage of an employed worker is:

$$E[w(x, y) | w \geq w_x^*] = E[w(x, y) | \underline{y}_x < y < \bar{y}_x] = \frac{1}{G(\bar{y}_x) - G(\underline{y}_x)} \int_{\underline{y}_x}^{\bar{y}_x} w(x, y) g(y) dy \quad (3.5)$$

## Unemployment

In this one-sided search environment, the expected unemployment duration is equal to the inverse of the hazard rate. For worker type  $x$ , the hazard rate is the probability of receiving an acceptable job offer in any period, which is equal to the probability of receiving an offer multiplied by the probability that the offer comes from a firm in the acceptable range.

$$\mathcal{H}_x = \lambda[1 - \tilde{G}(w_x^* | x)] = \lambda \int_{\underline{y}_x}^{\bar{y}_x} g(y) dy$$

The steady-state condition on unemployment requires that the change in the unemployment rate for each type of worker is equal to zero at all points in time.

$$\dot{u}_{x,t} = s(1 - u_t(x)) - u_t(x)\mathcal{H}_x = 0$$

$$u(x) = \frac{s}{s + \lambda \int_{\underline{y}_x}^{\bar{y}_x} g(y) dy} = \frac{s}{s + \lambda (G(\bar{y}(x)) - G(\underline{y}(x)))} \quad (3.6)$$



This function can be used to derive the distribution of unemployed workers. First, I define  $\bar{u}$  as the total measure of unemployed workers in the economy.

$$\bar{u} = \int_0^1 u(x) dL(x) \quad (3.7)$$

The cdf of unemployed workers is given by

$$F(x) = \frac{\int_0^x u(x) dL(x)}{\bar{u}} \quad (3.8)$$

Taking the derivative with respect to  $x$ , I obtain the pdf of unemployed workers,

$$f(x) = \frac{u(x)\ell(x)}{\bar{u}} \quad (3.9)$$

## 3.2 Computational Solution

Using the reservation wage equation (3.4), I provide a numerical example of the equilibrium. For this example, I use the following functional forms:

$$\rho(x, y) = \max\{x - \delta(x - y)^2, 0\} \quad b(x) = b_0 + b_1 \cdot x$$

Workers are distributed according to  $L(x) \sim \beta(\beta_1, \beta_2)$ , and the distribution of job vacancies  $G(y)$  is assumed to be the same as the worker distribution.

## Data

I use the 2009 to 2013 waves of the National Longitudinal Survey of Youth 1997 to calibrate several parameters for this example. The NLSY97 surveys individuals born between 1980-1984; during the time period used for this calibration, respondents are between the ages of 25-33 years old.

## Calibration

A time period in this model is taken to be one month. Calibrated parameter values used for the numerical example are presented in Table 1. For simplicity, workers are assumed to have no base value of leisure;  $b_0$  is set to 0. The monthly interest rate  $r$  is chosen to match the average 3-month treasury bill rate from 2009-2013. Other parameters are calibrated using data from the NLSY97 as described in this section.

I assume that worker and firm types are contained within the interval  $[0,1]$ , and use a beta distribution with parameters  $a$  and  $b$  to approximate the distribution of worker types.

Table 1: Calibrated parameter values

Parameter	Value	Target
$b_0$	0	
$b_1$	0.4	Monthly UI benefits/ avg. monthly income
$\beta_1$	2.1	ASVAB score distribution
$\beta_2$	2.4	ASVAB score distribution
$r$	0.001	3-month Treasury bill
$s$	0.0188	(Job duration) <sup>-1</sup>

This distribution will be used as the exogenous distribution of worker types  $L(x)$  for the computational exercises in this paper.

To calibrate the distribution of worker skills in the data, I use the ASVAB category scores, residualized by age and gender. I perform a linear transformation on the data so that the final variable lies in  $(0, 1)$ . Using maximum likelihood estimation to fit a beta distribution to the transformed scores, I arrive at the parameter values given in Table 1. Since the test scores used to create the distribution are designed to be normally distributed, the fitted distribution is very close to a symmetric normal distribution. However, distributions of other types of skills, such as leadership skills or communication skills, may be asymmetric. To illustrate how this impacts the model predictions, equilibrium results for two skewed distributions are provided in Appendix C.

To calibrate the separation rate, I use the inverse of the primary job duration<sup>15</sup>. Because some of the respondents in the NLSY97 may be just beginning their careers in 2009, I use only the oldest respondents (born in 1980) for this calculation; these individuals are ages 29-33 during the time period used. The average tenure on the primary job is 53.01 months, leading to a separation rate of  $s = \frac{1}{53.01} = 0.0188$ . For comparison, I calculate the same value using the NLSY79 data; respondents' average job tenure between 2009-2013 is 130.17 months, leading to a separation rate of 0.77% per month. However, the BLS reports that the average job duration in the U.S. is 4.6 years, so the individuals in the NLSY79 are a poor representation of the broader U.S. labor market in this respect.

The U.S. Department of Labor's Office of Unemployment Insurance releases a yearly Benefit Accuracy Measurement report containing each state's quarterly UI replacement rate. From 2009 to 2013, the weighted average U.S. replacement rate was between 0.405 and 0.470. To provide conservative estimates of  $\lambda$  and  $\delta$ , I calibrate the model using  $b_1 = 0.4$ .

<sup>15</sup>See Figure 18a in Appendix A for a histogram of job duration.

Model parameters that have not yet been calibrated are  $\lambda$  and  $\delta$ . I use the Simulated Method of Moments to select values for these parameters. I choose two moments to match, the hazard rate  $\mathcal{H}$  and the ratio of high-skill to median-skill log wage differentials,

$$\mathcal{D}_{90,10} = \frac{90^{th} \text{ percentile log(wage) (high-skill) / } 10^{th} \text{ percentile log(wage) (high-skill)}}{90^{th} \text{ percentile log(wage) (median-skill) / } 10^{th} \text{ percentile log(wage) (median-skill)}}.$$

Using the unemployment rate flow condition, I estimate the hazard rate from unemployment to employment as a function of the calibrated separation rate and unemployment rate. In particular, when considered in nearly continuous time, the unemployment rate follows

$$u_t = u_{t-1}(1 - \mathcal{H}_t) + s(1 - u_{t-1})$$

In a steady-state equilibrium,  $u_t = u_{t-1}$  and  $\mathcal{H}_t = \mathcal{H}$ . Solving this equation for the hazard rate, I set  $s = 1.88\%$  and use the average unemployment rate of  $8.35\%$  to calculate an average hazard rate of  $20.71\%$  per month. Using NLSY79 data, the calculated hazard rate is  $8.65\%$ .

To calculate  $\mathcal{D}_{90,10}$ , I use respondents' reported hourly compensation (including wages, salary, commissions, and tips)<sup>16</sup>. For each skill type, I calculate the 90/10 log wage differential

$$Diff = \frac{90^{th} \text{ percentile log(wage)}}{10^{th} \text{ percentile log(wage)}}.$$

I fit the log wage differentials using

$$\widehat{Diff} = \beta_0 + \beta_1 x + \beta_2 x^2$$

and use the fitted values to calculate the ratio of high-skill to median-skill log wage differentials,  $\mathcal{D}_{90,10}$ <sup>17</sup>.

Both the aggregate unemployment rate  $\bar{u}$  and the ratio of wage differentials  $\mathcal{D}_{90,10}$  can be matched individually by an infinite set of  $(\lambda, \delta)$  pairs. However, only one parameterization can match the two moments simultaneously<sup>18</sup>. Table 2 shows the values of the labor market moments from the data, as well as the values obtained after simulating the model using the estimated parameters  $\hat{\lambda}$  and  $\hat{\delta}$ . The unique  $(\hat{\lambda}, \hat{\delta})$  parameterization to be used in the calibrated model is given in Table 3.

<sup>16</sup>Since I compare  $\mathcal{D}$  to accepted wages from the simulated model, I drop those observations reporting an hourly wage of \$0 or less. I also drop individuals who worked less than 13 weeks in the year.

<sup>17</sup>Actual and fitted wage differentials are shown in Figure 18b in Appendix A.

<sup>18</sup>The sets of  $(\lambda, \delta)$  pairs that match each moment are plotted in figure 19 in Appendix A.

Table 2: Labor market moments obtained from data vs. those from the simulated model.

	Moment	Data	Simulated
Targeted	$\bar{u}$	8.35%	8.34%
	$\mathcal{D}_{90,10}$	1.0286	1.0295
Not Targeted	$\mathcal{H}$	20.71%	21.65%
	$\mathcal{D}_{90,50}$	1.0158	1.0144
	unemp. duration (months)	4.8296	4.8947

Table 3: Parameter values obtained using SMM

Parameter	Value	Target
$\hat{\lambda}$	0.2414	$\bar{u}, \mathcal{D}_{90,10}$
$\hat{\delta}$	0.1098	$\bar{u}, \mathcal{D}_{90,10}$

## Results

The exogenous distribution of vacancies  $G(y)$  gives rise to the distribution of potential wages conditional on the worker's type,  $\tilde{G}(w|x)$ , which is plotted in Figure 7. Because the mismatch parameter  $\delta$  is small relative to the maximum productivity, wage offer distributions for higher-skilled workers first-order stochastically dominate those for lower-skilled workers.

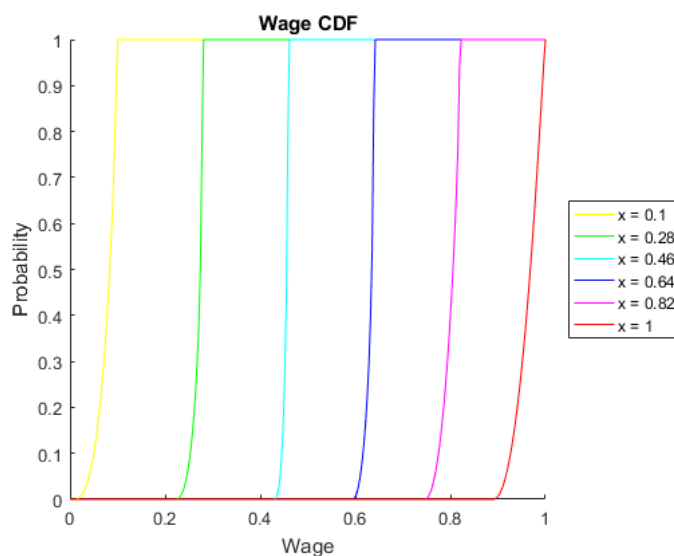


Figure 7: Wage CDF by Worker Type

To find the reservation wage for each type of worker, I iterate on equation 3.4 until the

function converges. The results are shown in Figure 8. The worker’s maximum wage,  $x$ , is plotted on the same graph for reference.

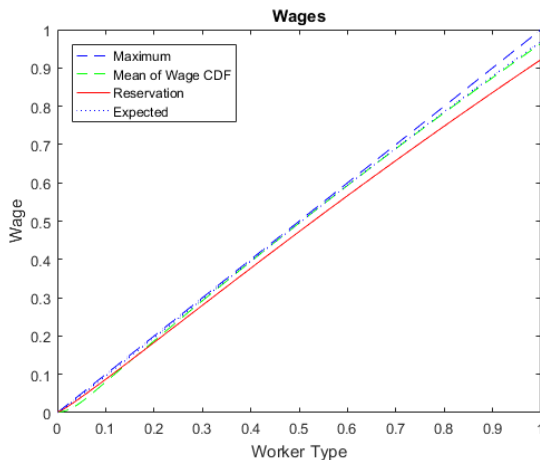


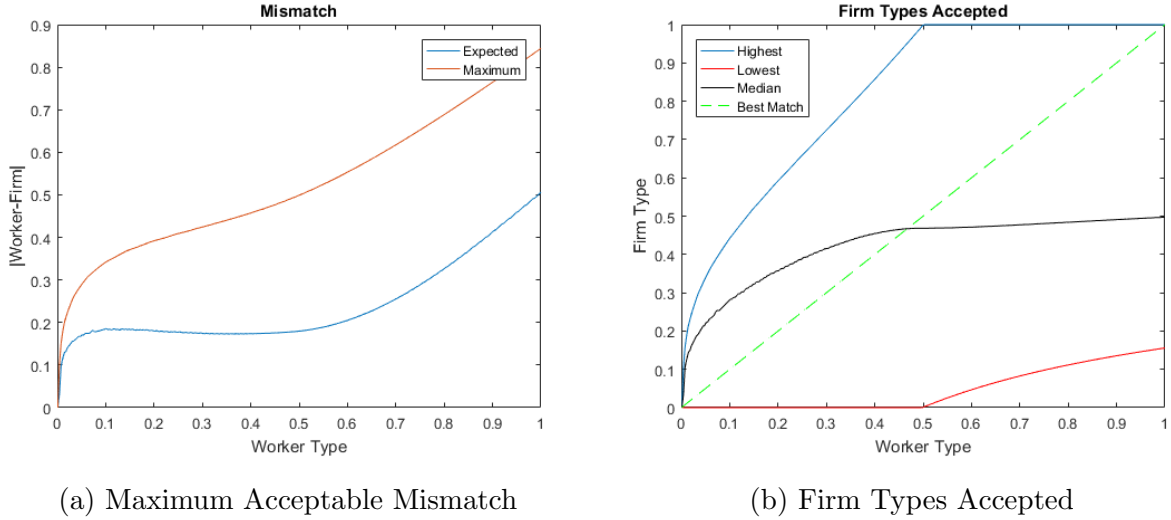
Figure 8: Comparison of expected, reservation, and mean wages.

It is clear that the reservation wage is increasing in the worker’s type. This result is somewhat intuitive, because better workers are more productive. However, the reservation wage is not linear. The slight curvature is due to the fact that it is relatively more difficult for workers near the ends of the distribution to find a job with low mismatch.

Figure 8 also shows the expected wage, conditional on being employed, for each type of worker. This is equal to the expected productivity in equation (3.5), following the initial assumption on wage setting. It is useful to note here that this is not equal to the mean of the worker’s wage distribution, since the reservation wage truncates the lower end of the distribution of wages for employed workers. For most workers, the mean wage and the expected wage are very close. However, workers near the low end of the skill space require a reservation wage higher than the mean of their wage distribution, so the expected wage is above the mean wage as well.

One of the questions that this model is designed to address is the level of mismatch accepted by workers. Specifically, do workers of different types accept different levels of mismatch? Because wages and mismatch are related one-to-one, the reservation wage can be used to answer this question. In this example, the level of mismatch associated with a wage  $w$  is given by  $|\mu| = 1 - \frac{w}{x}$ . Substituting in the reservation wage yields the maximum level of mismatch a worker of type  $x$  will accept. Figure 9a shows that the accepted level of mismatch does indeed vary by worker type. For workers near the upper end of the skill space, the difference between the  $E(x, w)$  and  $U(x)$  is quite large, even for wages near the low end of

Figure 9: Workers' mismatch acceptance strategies

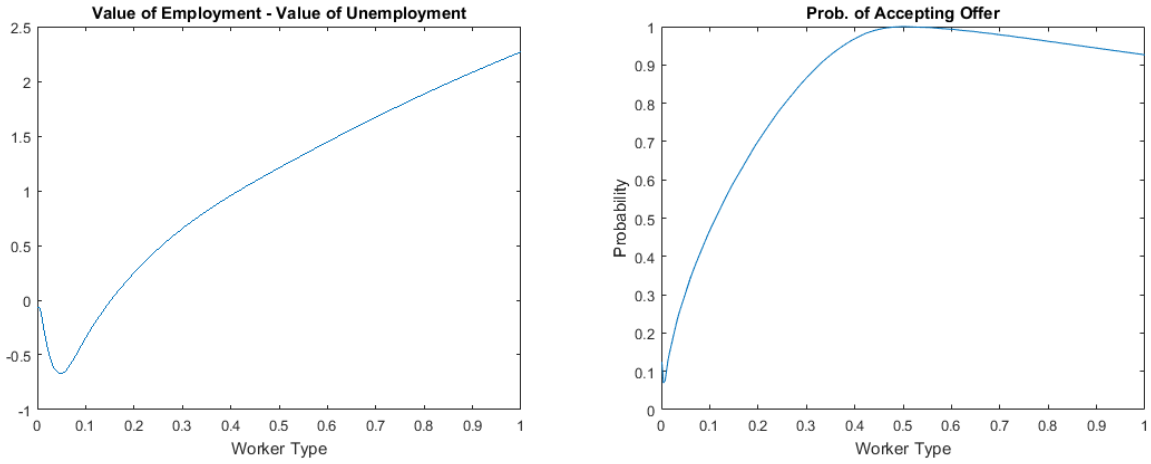


the conditional wage offer distribution. On the other hand, this difference is relatively small for low-skilled workers. Figure 10a shows the difference between  $E(x, w)$  and  $U(x)$ , setting  $w$  equal to the mean of the worker's wage offer distribution. Because low-skilled workers give up less by remaining unemployed, they are willing to wait longer for a better match. In fact, at the calibrated level of unemployment benefits, low-skilled workers are better off in unemployment than at a job that pays the mean wage offer. The range of firms accepted is symmetric around the line  $y = x$ ; workers accept job offers from all firms with mismatch less than or equal to  $|\mu^*| = 1 - \frac{w_x^*}{x}$ . The maximum and minimum firm types accepted by each type of worker are plotted in Figure 9b; all matches in between these two curves are accepted.

Conditional on receiving an offer, the probability that a worker of type  $x$  receives an acceptable job offer is equal to  $1 - \tilde{G}(w_x^*|x)$ . Both the wage distribution and the reservation wage vary across  $x$ , so the probability of an acceptable offer will also vary; the equilibrium acceptance probabilities are plotted in Figure 10b. The aggregate acceptance rate is 89.69%. Since the NLSY does not contain data on rejected job offers, it is not possible to directly compare the simulated acceptance rate to the data. However, the Survey of Unemployed Workers in New Jersey contains data on the receipt and acceptance of job offers for a sample of workers during 2009 and 2010; the average acceptance probability in this dataset is 79%.

This implies that the expected length of an unemployment spell will not be the same across all worker types. The unemployment duration is inversely related to the probability of an acceptable offer, so worker types with higher acceptance probabilities will have shorter

Figure 10: Opportunity cost of unemployment, and the proportion of offers accepted.



(a) Difference between the value of employment (setting wage equal to the mean wage) and the value of unemployment. (b) Probability of an acceptable wage offer.

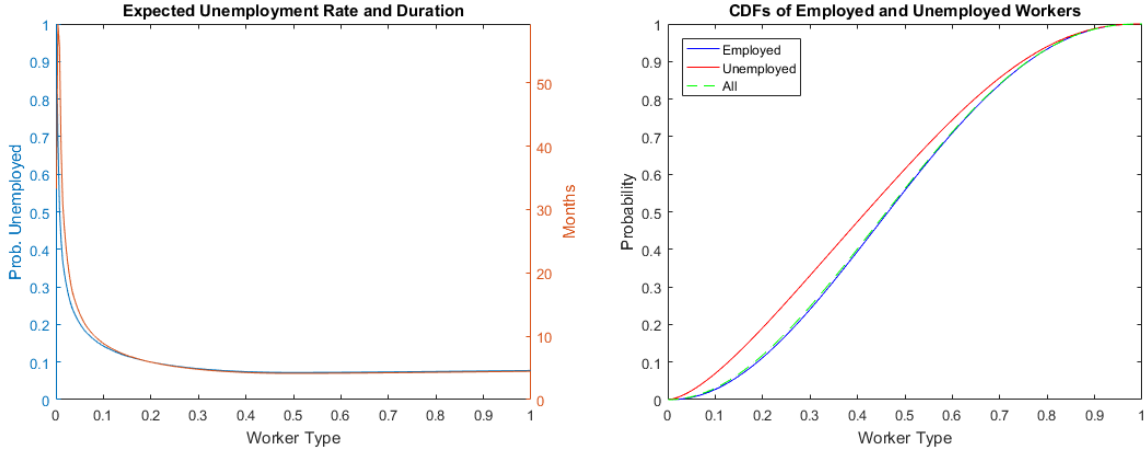
unemployment spells. The expected unemployment rate and duration for each type of worker is plotted in Figure 11a. A worker whose type is near the end of the distribution will wait much longer for a job than a worker whose type is near the center, even though the acceptable level of mismatch is similar for both workers. Therefore, the unemployment rate for low-skilled workers is higher than that of high-skilled workers. Because the unemployment rate is not constant for all types of workers, the distributions of employed and unemployed workers differ somewhat from the overall worker type distribution  $F(x)$ . While the shift in distribution is slight in this case, changes in the structure of unemployment benefits amplify the changes in the CDF shown here.

Comparable results using alternative parameterizations are given in Appendix C. While the results differ based on the calibration method used, there are no qualitative changes except in the case where unemployment benefits no longer depend on the worker’s skill type.

### 3.3 Discussion of Results

I assess the efficacy of this model by comparing the results from the calibrated model to the stylized facts in Section 2. Figure 12 compares the match acceptance sets observed in the data to those predicted by the model. To mitigate the effects of outliers in the data, the 95<sup>th</sup> and 5<sup>th</sup> percentiles of observed matches are used in place of the maximum and minimum acceptable matches. The model is effective at replicating the lower bound

Figure 11: Employment outcomes across worker types.



(a) Unemployment rate and duration.

(b) Distributions of employed and unemployed workers.

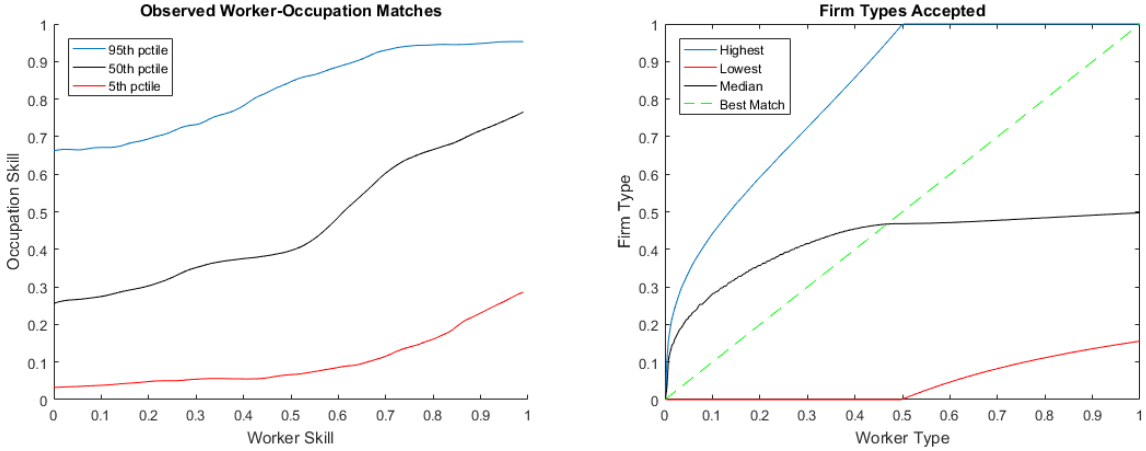
of match acceptance; on average, the model-predicted lower bound of match acceptance is 5.36% lower than the 5<sup>th</sup> percentile of matches observed in the data. For workers above  $x = 0.5$ , the model-generated result for the upper bound of match acceptance is only 7.87% above the 95<sup>th</sup> percentile of matches observed in the data on average. However, the model deviates from the data for workers in the lower half of the skill space; among this group, the average distance between the 95<sup>th</sup> percentile match and the predicted upper bound is 14.03%. The predicted upper bound for workers below the median skill is too low; these workers appear less selective in the data. However, the median match predicted by the model is more accurate for the lower half of workers; the model overshoots the median match by only 4.07% here. For workers above the median, the model underestimates the median match by 12.48% on average.

These results are reflected in the maximum and expected levels of mismatch for each worker type plotted in Figure 13. Among low-skilled workers, the model underestimates average mismatch by only 3.16%. Turning to the upper bound of mismatch, the model is unable to capture the long tail of higher-skilled jobs observed among these workers in the data. On the other hand, for workers above  $x = 0.5$ , the model predicts an average level of mismatch that is 8.14% above the average in the data, but successfully predicts the maximum level of mismatch accepted.

In line with the stylized facts, the model is able to generate an unemployment rate that is decreasing in the worker's type; the comparison of unemployment rates is shown

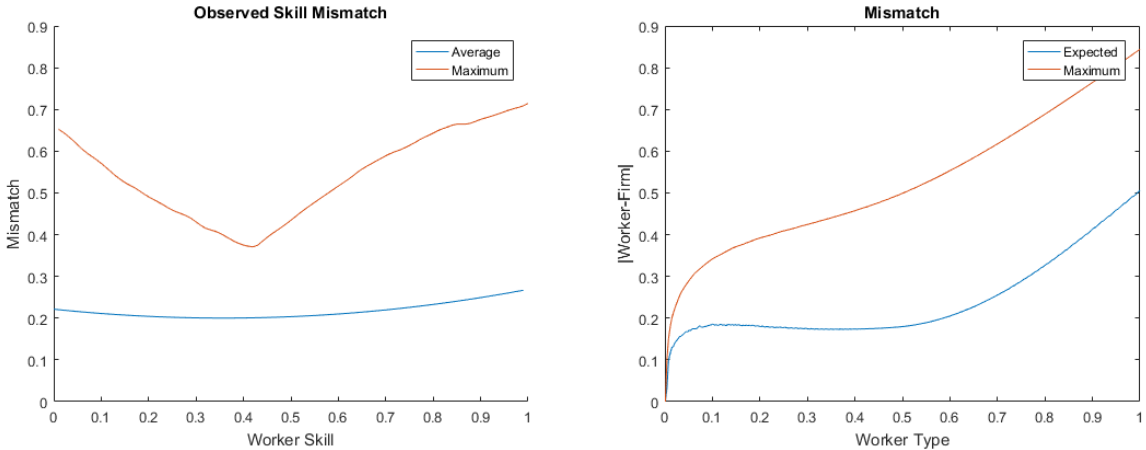


Figure 12: Match acceptance sets.



(a) 95th, 50th, and 5th percentiles of matches observed in the NLSY97. (b) Maximum, median, and minimum firm types accepted in the calibrated model.

Figure 13: Mismatch in the data and calibrated model.

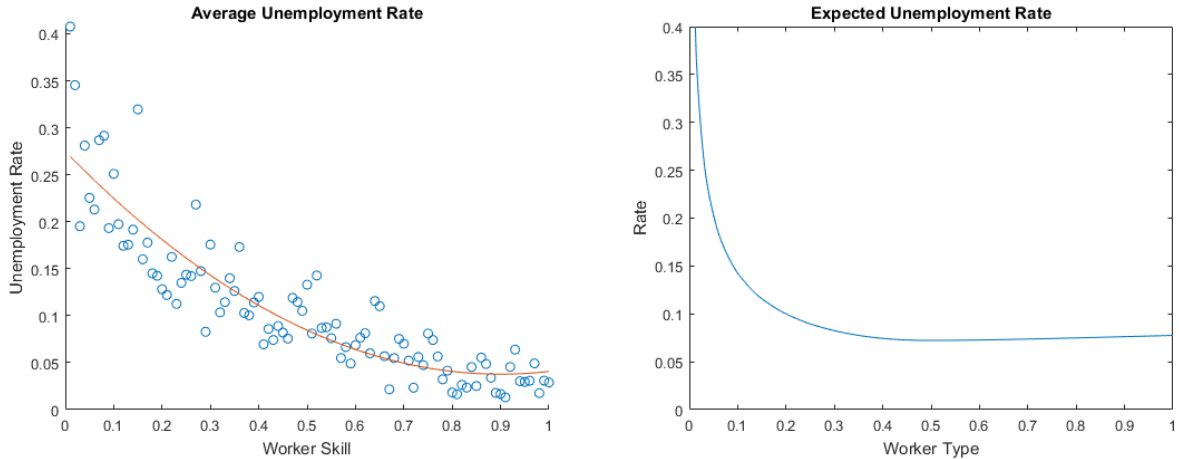


(a) Average and maximum mismatch observed in the NLSY97. (b) Expected and maximum mismatch accepted in the calibrated model.

in Figure 14. Among workers above  $x = 0.5$ , the model-generated unemployment rate is on average 2.45% greater than that in the data. For workers below the median, it is 4.7% too low on average. The model prediction for the expected duration of unemployment is also broadly consistent with the empirical facts; higher-skilled workers are expected to exit unemployment more quickly.

The comparisons in this section highlight several similarities between the stylized facts and the model predictions, as well as two distinct areas of divergence. The model predicts too

Figure 14: Unemployment rates by worker skill type.



(a) Unemployment rates in the NLSY97. (b) Unemployment rates in the calibrated model.

much selectivity among low-skilled workers, yet it is able to generate an accurate prediction of the median match. In contrast, the range of matches accepted by high-skilled workers is successfully replicated, but the model predicts a median match that is well below that in the data. Together, these two facts suggest that workers are able to sort more effectively than is allowed by the model. While random search provides a simple and tractable way to generate non-singleton match sets, the results here suggest that workers' search is not perfectly random; the implications of a partially-directed search strategy will be explored in subsequent work.

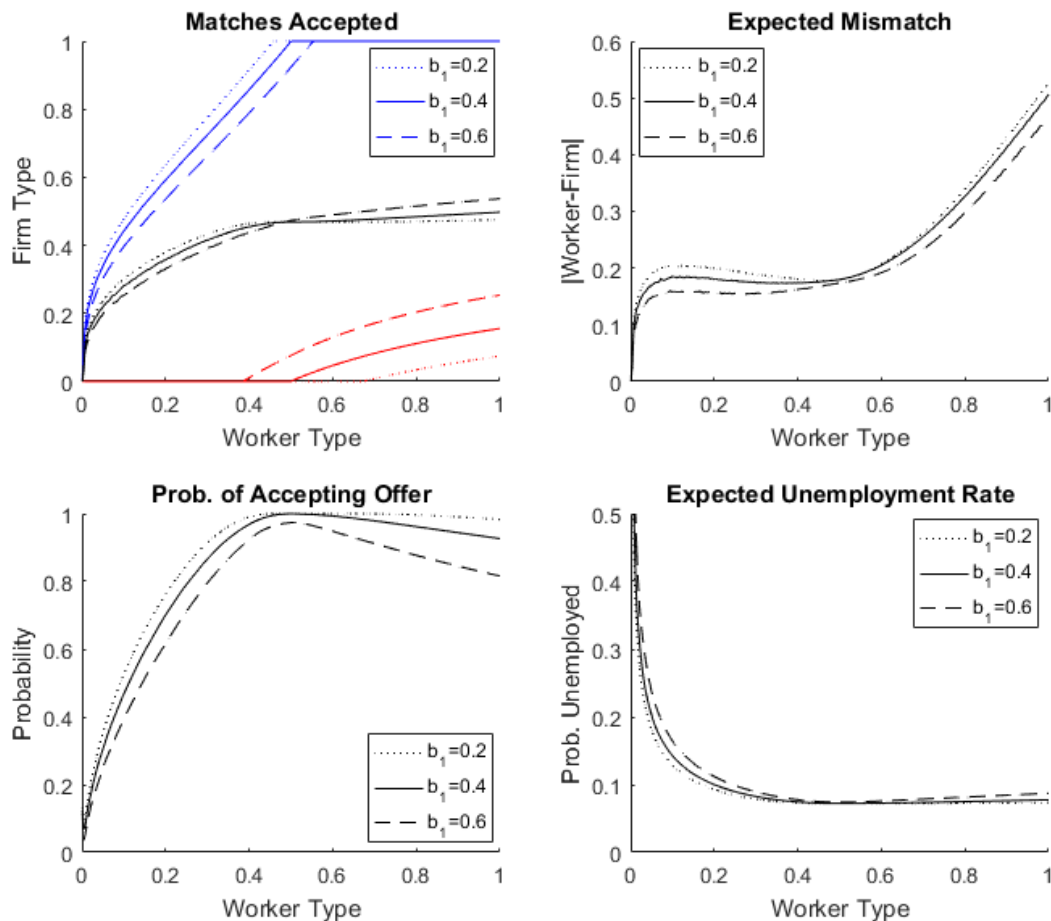
## 4 Unemployment Insurance and Skill Mismatch

Can unemployment insurance (UI), through its role as a subsidy for job search, decrease aggregate skill mismatch in the labor market? Here, “aggregate skill mismatch” refers to the sum of pair-specific skill mismatch across all accepted worker-firm matches. Previous literature has shown that unemployment insurance can act as a search subsidy, allowing workers to remain unemployed longer in order to wait for a better offer. However, the impact of UI on mismatch, wages, and unemployment in a model with heterogeneity is not clear. Because it allows for unemployment insurance benefits to vary across worker skill types, the present model provides a framework to examine the labor market impacts of different unemployment insurance schemes. The counterfactuals in this section employ the same parameterization as the previous section, changing the UI parameters  $b_0$  and  $b_1$ . In

each case, I compare the social welfare under the new policy to the value of  $\Omega$  obtained from the calibrated model.

To provide a comparison with the calibrated value of  $b_1 = 0.4$ , I begin this section with two alternative policies that vary the replacement rate,  $b_1 = 0.2$  and  $b_1 = 0.6$ . The results are plotted in Figure 15. As expected, workers' match acceptance sets widen as unemployment benefits decrease, and narrow when benefits increase. The expected mismatch, acceptance probabilities, and unemployment rates react accordingly. Social welfare in the low UI case is 1.7% lower than the calibrated model, while in the high UI case welfare is 1.7% higher. Under the social welfare function defined in this paper, high (low) UI benefits make all workers better (worse) off; the UI policy has no redistributive properties.

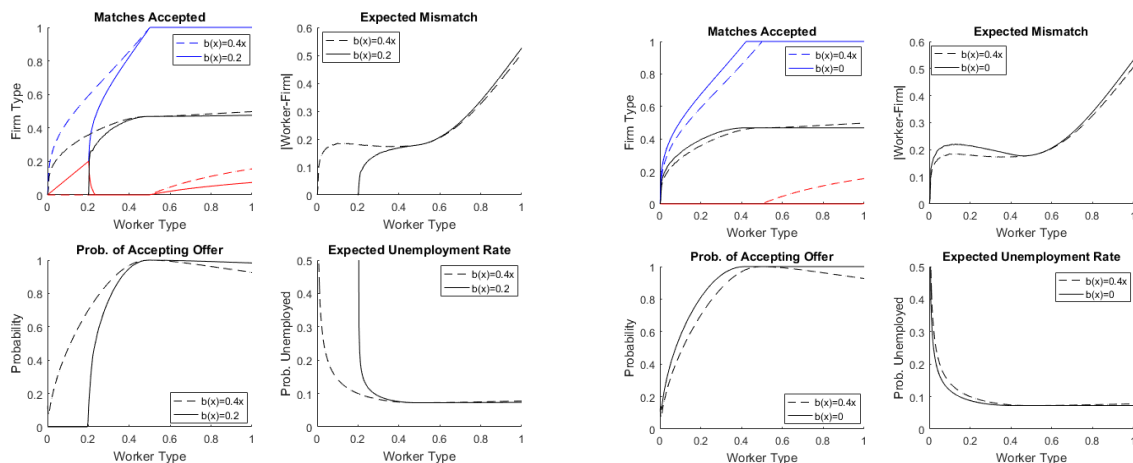
Figure 15: High vs. low replacement rate



A laissez-faire economy with no unemployment benefits ( $b_0 = b_1 = 0$ ) provides a

baseline for the counterfactual policy analysis. As expected, welfare in an economy with no unemployment benefits is lower than in the calibrated model; social welfare here is 3.3% below that of the calibrated model, and again all workers are made worse off. Figure 16a plots the workers' acceptance sets, expected mismatch, probability of receiving an acceptable offer, and expected duration of unemployment. At the preferred values of  $\delta$  and  $\lambda$ , the wage penalty for mismatch is relatively small and offers are sufficiently infrequent that most unemployed workers are willing to accept a match with any type of firm. As in the main parameterization, increasing either of these parameters causes workers' acceptance sets to narrow, reducing mismatch. However, workers whose value of employment is sufficiently low still reject some high-mismatch offers. The simulated acceptance probability rises to 94%, the unemployment rate drops to 7.85%, and the average duration of unemployment falls to 4.55 months.

Figure 16: Constant unemployment benefits



(a) Zero unemployment benefits

(b) Positive unemployment benefits ( $b_0 = 0.2$ )

Fixing unemployment benefits at a constant value across all worker types provides a useful comparison to previous results in the literature. I set  $b_0 = 0.2$ , the value of benefits for a worker with type  $x = 0.5$  under the preferred UI scheme. Social welfare remains approximately 1.2% lower than the calibrated model; the welfare of low-skilled workers increases compared to the calibrated model, and that of high-skilled workers decreases. The graphical results are shown in Figure 16b. Workers with types  $x < 0.2$  do not work; even at the ideal match, their wages are less than the unemployment benefit payment. Workers whose skill type is above this cutoff employ strategies similar to those in the main model.

The aggregate unemployment rate jumps to 19.43%; the unemployment duration for workers with types  $x < 0.2$  is infinite, and the average duration for workers with  $x > 0.2$  falls to 4.69 months.

Finally, I explore whether a UI policy with a varying replacement rate can decrease mismatch compared to the calibrated model. Specifically, I implement a replacement rate that increases in the worker's skill type; this encourages high-skilled (and frequently highly mismatched) workers to be more selective, while retaining an incentive for low-skilled workers to accept matches. The UI policy function used in the example in Figure 17 is:

$$b(x) = (0.2 + 0.4x) \cdot x$$

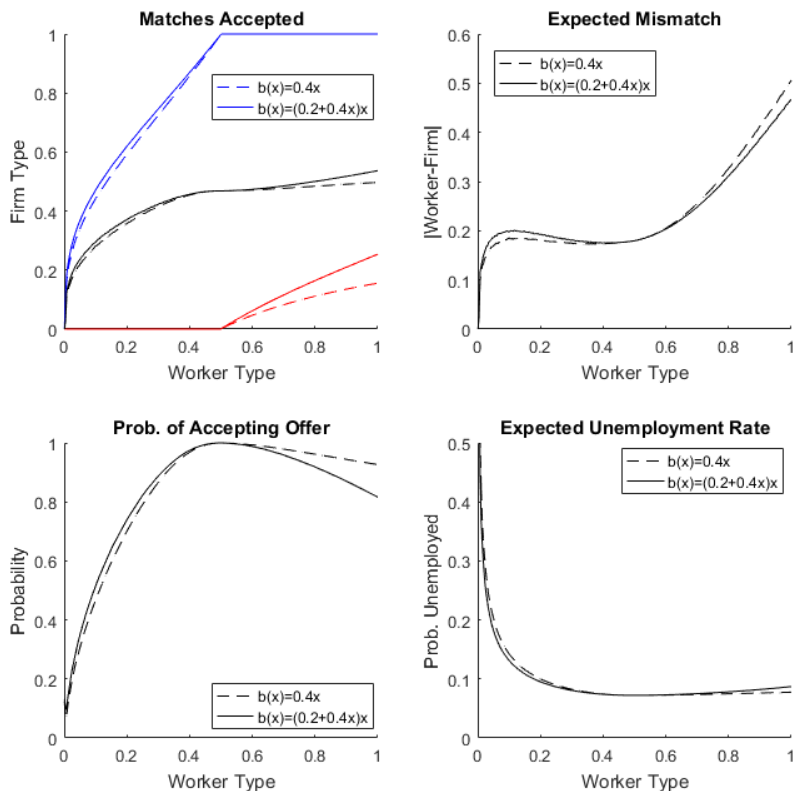
Under this policy, a worker of type  $x = 0.5$  again receives unemployment benefits equal to 40% of his maximum wage. In this economy, social welfare is only 0.2% greater than the calibrated model; high-skilled workers are better off under this UI policy, but low-skilled workers are worse off. Low-skilled workers accept a few more job offers, and high-skilled workers are noticeably more selective. The average acceptance probability increases to 89.96%, the aggregate unemployment rate decreases to 8.23%, and the average unemployment duration is 4.81 months.

Comparing the counterfactual UI policies in this section, the economy with a high replacement rate of 60% for all workers yields the greatest social welfare. However, the social welfare function defined in this paper is not able to account for the cost of the UI policy, so it is not clear that a constant and high replacement rate is in fact the optimal policy. A revenue-neutral UI policy may also have significant redistributive effects, particularly since the workers' expected unemployment rates will change along with the value of unemployment. The question of optimal UI policy design in this model presents a challenge, because the social planner must take into account the impact of the UI policy on the workers' match acceptance strategies when choosing a policy structure. The design of an optimal UI policy will therefore be explored in a follow-up paper.

## 5 Conclusion

In this paper, I provide new evidence on the presence of skill mismatch in the labor market, and construct a one-sided search model to explain workers' strategies relating to the acceptance of skill mismatch. Using data from the NLSY97, I document three stylized facts relating to worker skill heterogeneity and skill mismatch. First, I show that higher-skilled

Figure 17: Replacement rate increasing in skill



workers are less likely to be unemployed, experience shorter unemployment spells, and earn higher incomes. This comes as no surprise, and confirms that the measure of worker skills used in this paper is indeed capturing a salient characteristic. Second, I find that the 90-10 and 90-50 wage differentials are positively correlated with the worker's skill type. This suggests that each type of worker can be employed in a range of jobs, and that higher-skilled workers may be willing to match with a wider range of occupations. Finally, I demonstrate that this is indeed the case; for each type of worker, I construct the acceptance region in the firm type space. There is some evidence for positive sorting, but matching is not perfect. Higher-skilled workers match with better jobs on average, but these workers also tolerate relatively more mismatch in order to exit unemployment. This novel fact motivates the creation of a search model with skill mismatch, in order to better understand workers' strategies.

For the purpose of this paper, I focus only on the worker's problem. I augment the McCall (1970) model to allow for heterogeneity among both workers and firms, as well as a match-specific productivity function that is decreasing in the level of pair-specific mismatch. The inclusion of worker types into the McCall model does not affect the basic structure of

workers' value functions. Though the value functions now depend on the worker's type, the mechanisms remain unchanged. This suggests that the predictions of this model should be comparable to those of the McCall model, and indeed that is what I find. Workers' decisions are summarized by a reservation wage, and that reservation wage depends on the flow value of unemployment, as well as the parameters representing the offer arrival rate, separation rate, and discount rate. As with the McCall model, the reservation wage is increasing in the level of unemployment benefits. However, moving to a distribution of worker types adds another dimension to the reservation wage analysis. Because each type of worker faces a different wage distribution, the reservation wage strategy now depends on the worker's type in a nonlinear way. As a result, the predicted unemployment rate and duration decrease nonlinearly in the worker's type.

Additionally, this model addresses mismatch in a way that the McCall model is unable to do. As one might expect, the level of mismatch accepted varies with worker type. Specifically, the model predicts that higher-skilled workers display increased tolerance for mismatch. Comparing workers' expected value of employment to the value of unemployment sheds light on this counterintuitive finding; more skilled workers face a larger disparity between the value of employment to that of unemployment. This provides a strong incentive to quickly exit unemployment, leading higher-skilled workers to be less selective in accepting job offers.

The calibrated model predicts match sets, unemployment rates, and expected unemployment durations that are broadly consistent with the stylized facts. However, the results are inconsistent with the data in two cases. Relative to the data, low-skilled workers in the model reject too many jobs for which they are underqualified, and high skilled workers experience too much mismatch on average. This brings the assumption of a random search strategy into question; despite accepting a wide range of matches, workers in the data are able to sort more effectively than the model allows. I suggest that this may be reconciled by allowing workers to imperfectly target their search; this hypothesis will be explored in subsequent work.

Finally, I assess the effect of unemployment benefits on workers' match acceptance decisions. I show that UI policies affect workers' strategies in predictable ways; higher UI benefits make workers more patient, narrowing the match acceptance regions and reducing mismatch. This raises the question of whether a social planner might find it optimal to reduce mismatch (and increase aggregate productivity) through the provision of a UI policy that varies across worker types. For example, the planner may prefer high-skilled (and therefore

highly-productive) workers to be more selective, while encouraging low-skilled workers to exit unemployment quickly. I provide suggestive evidence that this may indeed be the case; however, the design of optimal UI policy is beyond the scope of the current paper.



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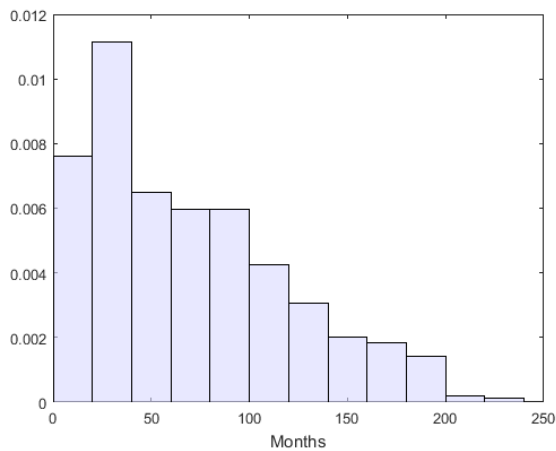
# A Data

## A.1 Descriptive statistics for NLSY97 and O\*NET

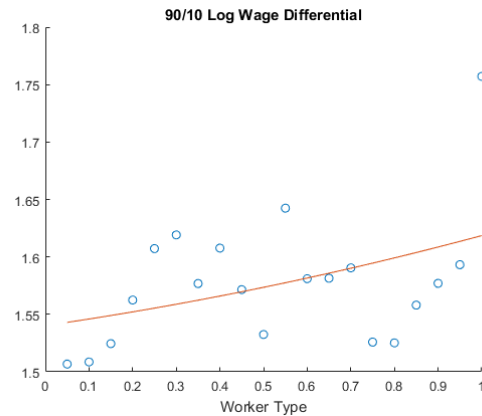
Table 4: Descriptive statistics of the cross-sectional sample, NLSY97, 2009-2013

Description	Statistic
Highest degree:	
None	5.32%
GED	10.13%
High School	41.09%
Associates	8.94%
Bachelors	24.77%
Masters	7.88%
PhD or Professional	1.59%
Job tenure (mean)	2.73 years
Unemployment duration (mean)	19.36 weeks
Weeks worked per year (mean)	45.29 weeks

Figure 18: Additional graphics.



(a) Histogram of job tenure.



(b) 90/10 log wage differential by ACT score.

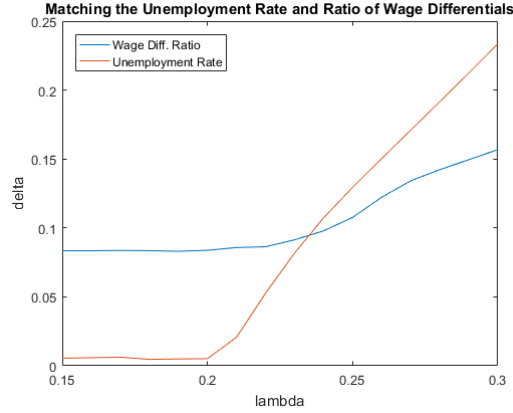


Figure 19: Matching the aggregate unemployment rate and the ratio of wage differentials in  $(\lambda, \delta)$  space.

## A.2 Calibration

Table 5 summarizes the range of values each parameter or labor market moment takes on through all alternative calibration methods, as well as the value from the preferred parameterization.

Table 5: Minimum, maximum, and preferred parameter values for the calibrated model.

Parameter	Min	Max	Preferred
$b_0$	0	0	0
$b_1$	.2003	.465	.4
$r$	.001	.001	.001
$s$	.0077	.0188	.0188
$\lambda$	.0887	.4	.225
$\delta$	0.0863	.41	.1264

Moment	Min	Max	Preferred
$\bar{u}$	.0816	.087	.0835
$\mathcal{H}$	.0864	.2084	.2071
$\mathcal{D}_{90,10}$	1.0255	1.0331	1.0331
$\mathcal{D}_{90,50}$	1.0006	1.0158	1.0158
Prob(accept)	.685	.9537	.7922

## B Robustness Checks

### B.1 Alternative definitions of skill type

Tables 6 and 7 display the  $R^2$  of the following regression, for various definitions of worker and occupation skill types. For each definition, the final  $x$  score is the individual's percentile rank.

$$H_i = \beta_0 + \beta_1 z_i + \beta_2 z_i^2$$

In the above regression,  $z_i$  is the skill type percentile (from 1 to 100, in increments of 1) and  $H_i$  is hourly compensation .

Table 6: Varying definitions of worker skill type.

$x$	Ranking definition	$R^2$
x_pca	PCA of individual's aggregated ASVAB score and highest degree	0.0619
x_pcawgt	PCA of scores in 4 math and verbal ASVAB categories and highest degree; scores residualized by race, gender, and 3-month age cohort (preferred method)	0.0772
x_act	For individuals with high school diploma or less, aggregated ASVAB score. For others, regress $ASVAB_i = \beta_0 + \beta_1 ACT_i + \beta_2 ACT_i^2 + \epsilon_i$ on scores of all individuals with ACT scores, use predicted ASVAB score as $x$ value.	0.0534
x_stack	Rank by education level, and within education level rank by aggregated ASVAB score.	0.0732
x_weight	$x_i = p \cdot ASVAB_i + (1 - p) \cdot \overline{ASVAB}_i$ , where $p = 0.8$ , $ASVAB_i$ is respondent's aggregated ASVAB score, and $\overline{ASVAB}_i$ is average ASVAB score of respondents with same education level as $i$ .	0.0551
x_optwgt	Same as above, except $p$ is chosen to minimize aggregate mismatch: $p^* = \underset{p}{\operatorname{argmin}} \left\{ \sum_{t=2009}^{2013} \sum_{k=1}^2 \sum_{i=1}^N x_i - y_{i,k,t} \right\}$ where $i$ is an individual, $t$ is the year, $k$ is a job, and $y_{i,k,t} = y_{i,jdm}$ . $p^* = .143$ ; there is a small amount of overlap between the top of one education category and the bottom of the next.	0.0743

Table 7: Varying definitions of occupation skill type.

$y$	Ranking definition	$R^2$
y_ct	Critical Thinking	0.0936
y_cps	Complex Problem Solving	0.1088
y_jdm	Judgment and Decision Making	0.1091
y_cpsjdm	PCA of Complex Problem Solving, Judgment and Decision Making	0.1152
y_jdmtech	PCA of Judgment and Decision Making, Repairing	0.0752
y_cogsk	PCA of all O*NET skill descriptors listed as “cognitive”	0.1129
y_cogab	PCA of all O*NET ability descriptors listed as “cognitive”	0.1109
y_tech	PCA of all O*NET skill descriptors listed as “technical”	0.0072
y_phys	PCA of all O*NET ability descriptors listed as “physical”	0.0419
y_social	PCA of all O*NET skill descriptors listed as “social”	0.0552

This comparison motivates the choice of  $x_{pcawgt}$  and  $y_{cpsjdm}$  as the preferred methods of ranking individuals and occupations, since these rankings best predict average hourly wages.

## B.2 “Similar” workers with different jobs

For  $x$  to be a good measure of worker skills, we would like to see that characteristics of workers with the same  $x$  do not differ systematically across occupations. Figure 21 shows the average age, average birth year, proportion male, and proportion white for workers of similar skill types (deciles of  $x$ ) in different occupations (deciles of  $y$ ). Each connected line represents workers in the same decile who are employed in occupations no more than two deciles away from their own skill type (since the number of observations in a category decreases substantially beyond this point). It is clear that  $x$  fully accounts for the individual’s education; Figure 20a shows that, conditional on  $x$ , individuals do not systematically sort into occupations based on their education level. The other three panels of Figure 21 suggest that, while there is substantial variation in the characteristics of similar workers across different occupations, workers are not sorting in any consistent way by age, gender, or race.

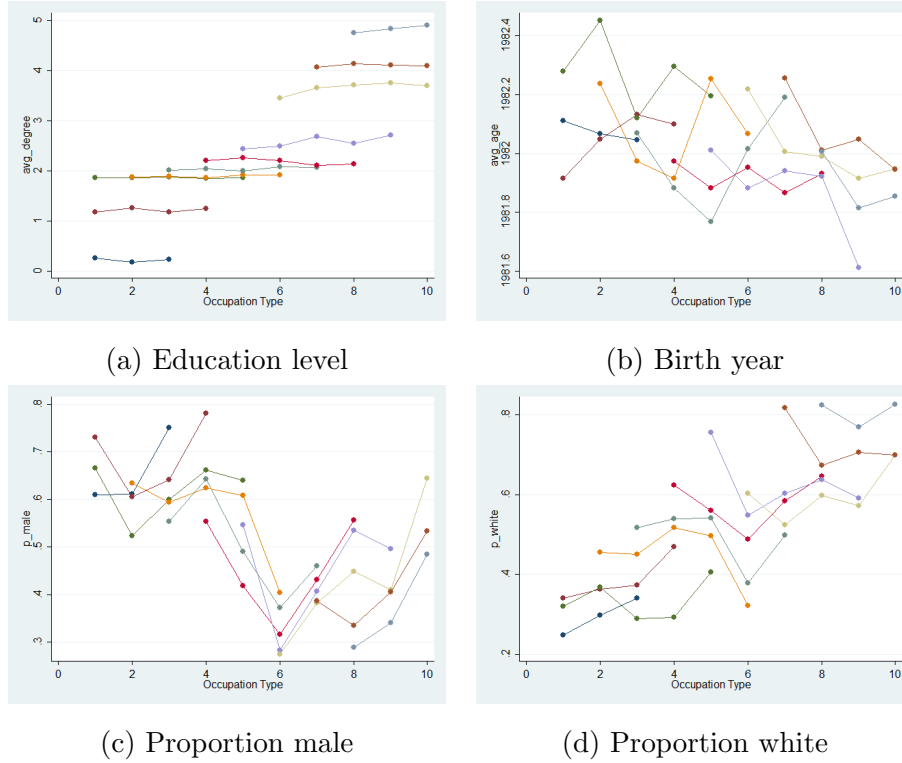
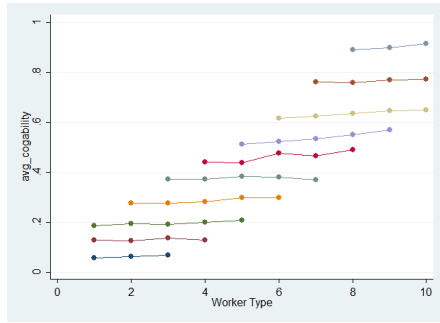


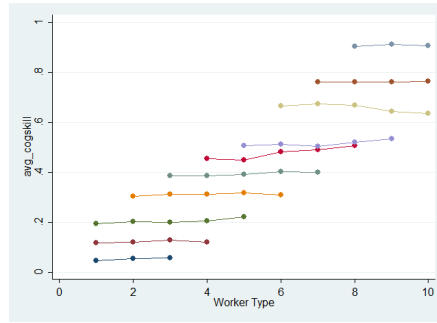
Figure 20: Characteristics of similar workers in different occupations.

### B.3 “Similar” occupations employing different workers

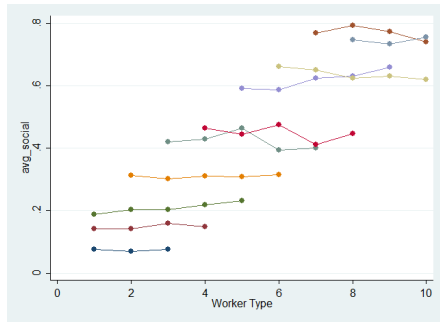
Analogous to the previous section, a good measure of occupation skills would capture all of the characteristics by which workers sort into occupations. Figures 21a through 21d show that the measure of occupation skills is indeed capturing the sorting of workers according to cognitive skills and abilities, social skills, and wages. However, Figures 21e and 21f suggest that there remains sorting of workers on other dimensions. After controlling for the occupation skill type (decile of  $y$ ), workers with higher cognitive skills (by decile of  $x$ ) sort into occupations that require lower levels of physical abilities and technical skills. This is not entirely unexpected, since the method of ranking skills was chosen to measure only cognitive skills. To attempt to correct for this, I included Repairing, a skill highly correlated with other technical skills and physical abilities, in a principal components analysis and recomputed the rankings of  $y$ . However, under this new ranking, jobs of the same type employing different types of workers systematically varied in wages and cognitive skill requirements. Since the object is to measure rank occupations by cognitive skills, the original measure is more favorable.



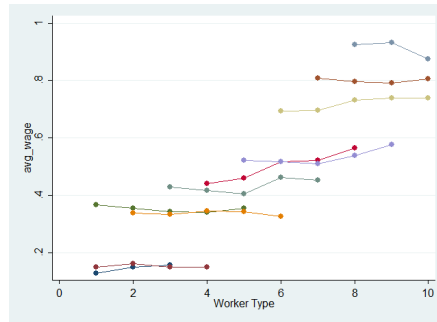
(a) Cognitive abilities



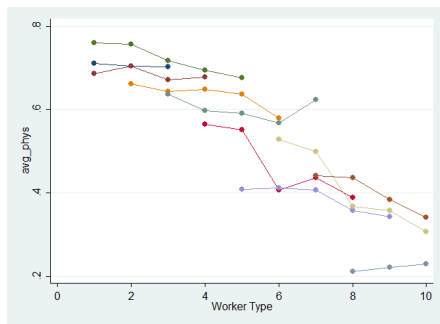
(b) Cognitive skills



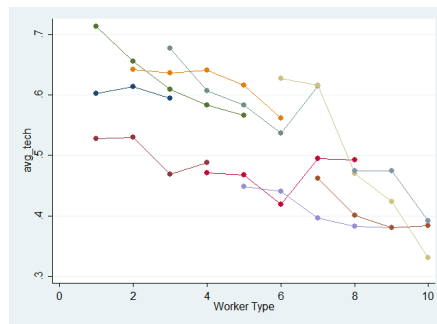
(c) Social skills



(d) Wage



(e) Physical abilities



(f) Technical skills

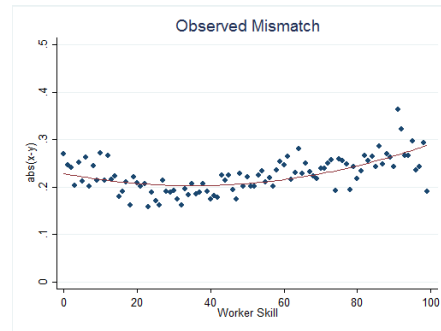
Figure 21: Characteristics of similar occupations employing different workers.



## B.4 Results by Gender



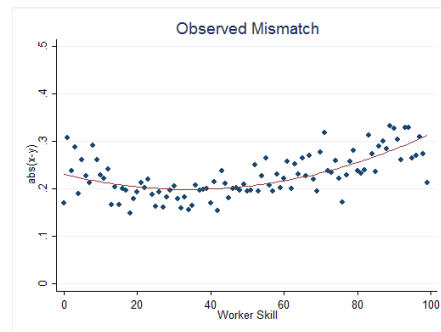
(a) Match sets (males only)



(b) Mismatch (males only)



(c) Match sets (females only)

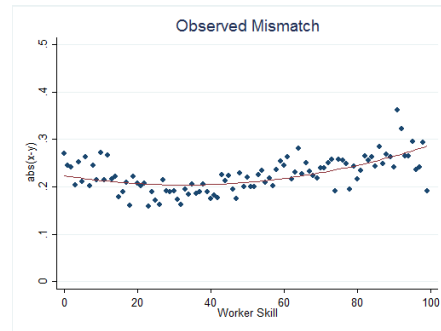


(d) Mismatch (females only)

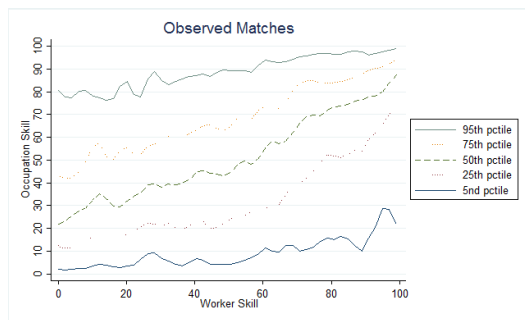
## B.5 Results by Race



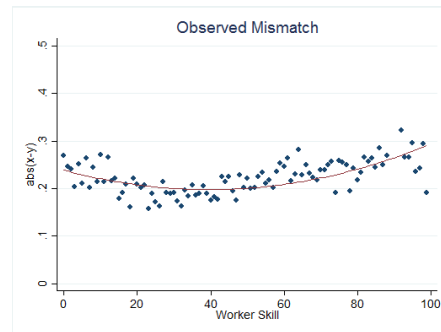
(a) Match sets (white only)



(b) Mismatch (white only)



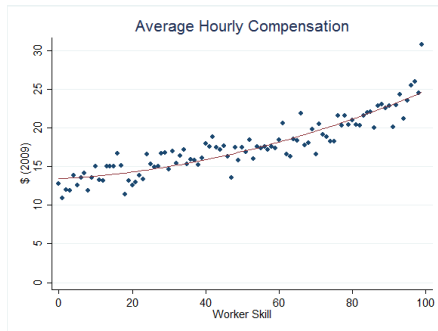
(c) Match sets (black only)



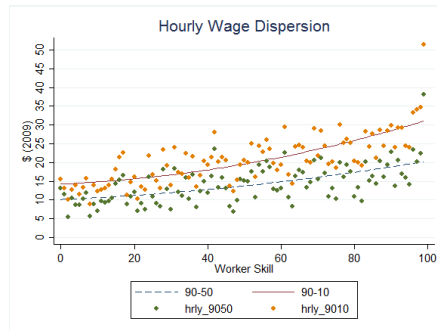
(d) Mismatch (black only)

## B.6 Alternative $x$

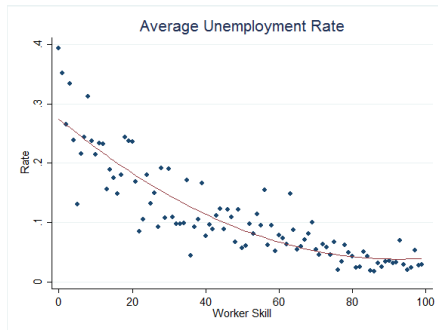
In this section, `x_optwgt` is used.



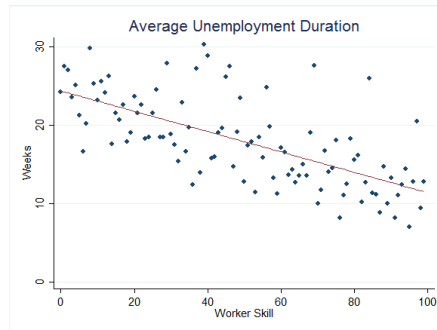
(a) Average wage



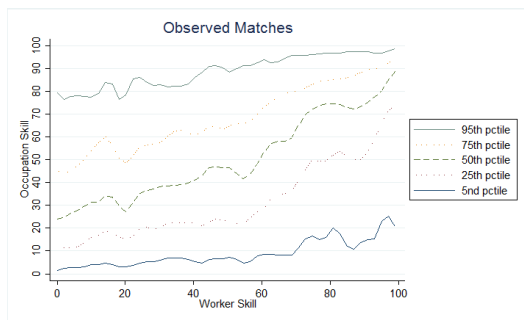
(b) Wage dispersion



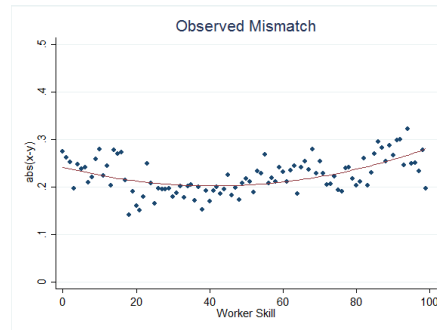
(c) Unemployment rate



(d) Unemployment duration



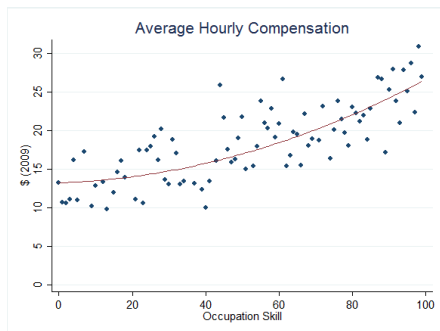
(e) Match sets



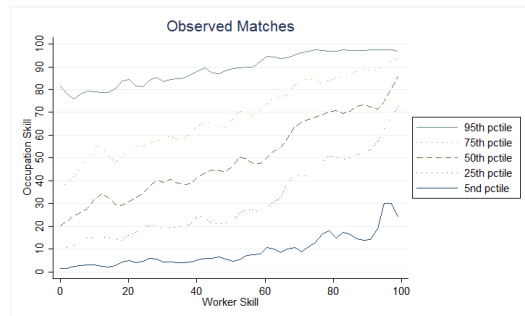
(f) Mismatch

## B.7 Alternative $y$

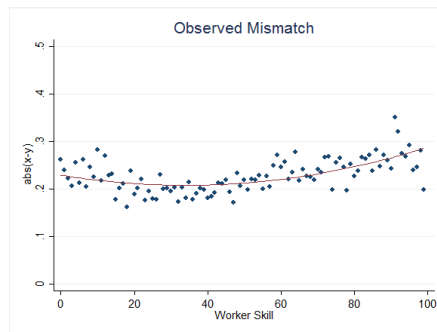
In this section,  $y_{\text{cogskill}}$  is used.



(a) Average wage by occupation



(b) Match sets



(c) Mismatch

## C Alternative Calibration Methods

In this section, I present the decentralized equilibrium results using several alternative calibrations, outlined in Table 8. Equilibrium results are plotted in Figures 26 through 29, and Table 9 summarizes the match between the model and the moments calculated from the NLSY97 data.

Table 8: Equilibrium results are provided for the following alternative parameterizations.

Parameter	(1)	(2)	(3)	(4)
$b_0$	0	0	0	0
$b_1$	0.4	0.3251	0.4	0.4
$r$	0.001	0.001	0.001	0.001
$s$	0.0188	0.0188	0.0148	0.0188
$\lambda$	0.2211	0.2188	0.1760	0.2541
$\delta$	0.1224	0.1113	0.1219	0.1714
Targets	$\mathcal{H}, \mathcal{D}_{90,10}$	$\bar{u}, \mathcal{D}_{90,10}$	$\bar{u}, \mathcal{D}_{90,10}$	$\mathcal{D}_{90,10}$

Alternate calibration (1), shown in Figure 26, targets the hazard rate rather than the unemployment rate when using SMM to calibrate  $\lambda$  and  $\delta$ . The hazard rate and unemployment rate are directly related, so only one can be used at a time for SMM. All other parameters are unchanged.

Alternate calibration (2), shown in Figure 27, uses the unemployment insurance replacement rate calculated from the NLSY97 data. The unemployment rate and ratio of wage differentials are used in SMM as targets to calibrate  $\lambda$  and  $\delta$ . All other parameters are unchanged.

Alternate calibration (3), shown in Figure 28, uses the lower separation rate calculated from the NLSY97 weekly employment arrays. The unemployment rate and ratio of wage differentials are again used in SMM as targets to calibrate  $\lambda$  and  $\delta$ , and all other parameters are unchanged.

Alternate calibration (4), shown in Figure 29, uses the job offer arrival rate  $\lambda$  calculated from the NJUI 2009 data. The ratio of wage differentials is used in SMM as a target to calibrate  $\delta$ , and all other parameters are unchanged.

Taken together, the four alternative parameterizations show a clear pattern. Changes in  $\lambda$  and  $\delta$  balance each other out. When  $\lambda$  increases, workers are able to exit unemployment more quickly; however, a corresponding increase in  $\delta$  causes workers to reduce the range of

Figure 26: Decentralized equilibrium results for alternative parameterization (1).

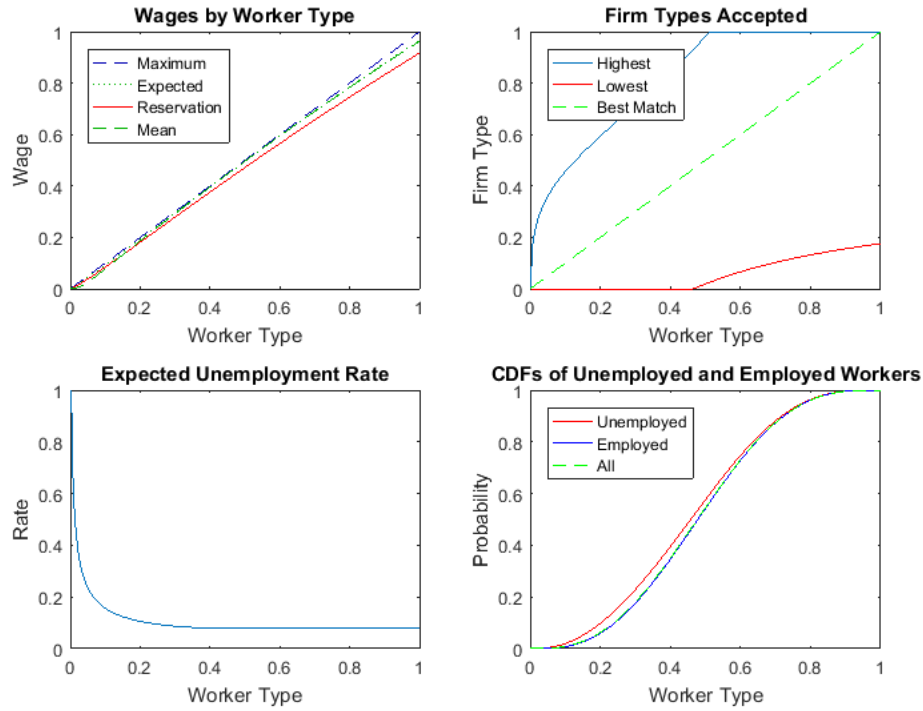


Figure 27: Decentralized equilibrium results for alternative parameterization (2).

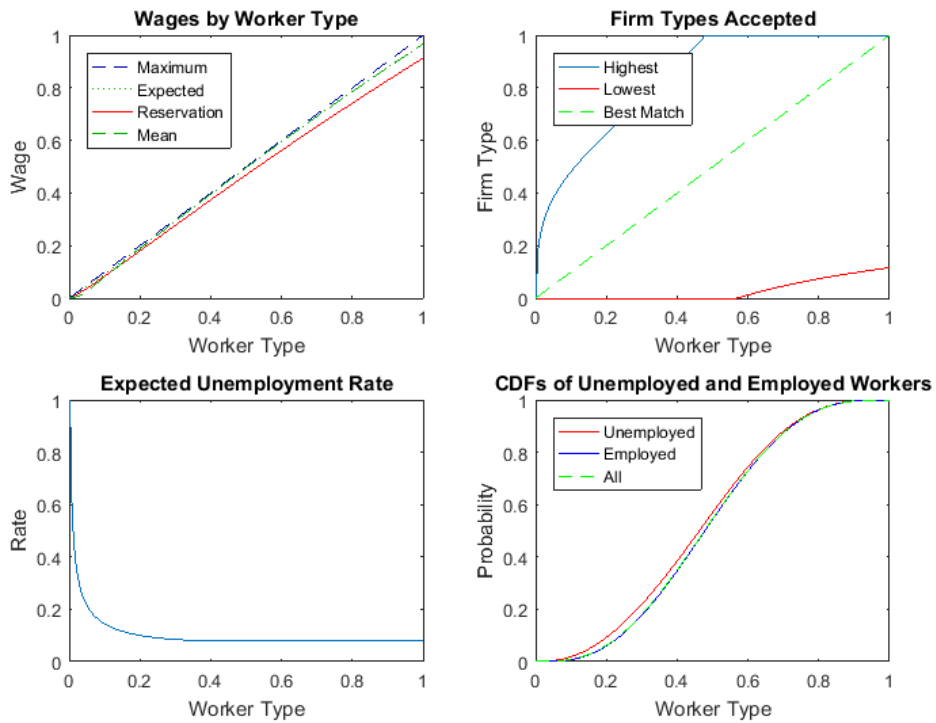


Figure 28: Decentralized equilibrium results for alternative parameterization (3).

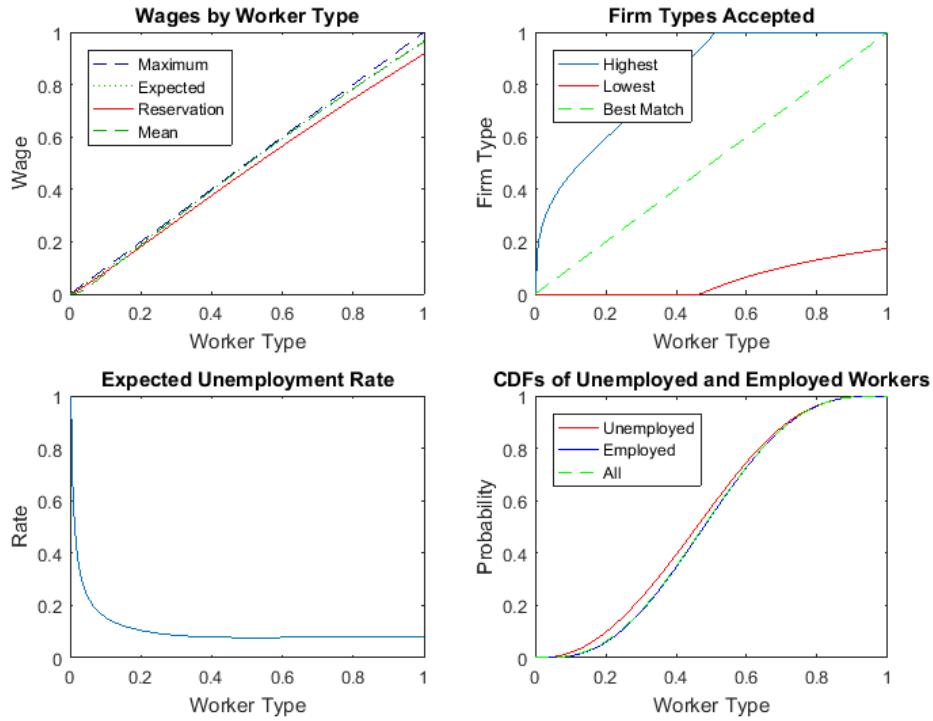
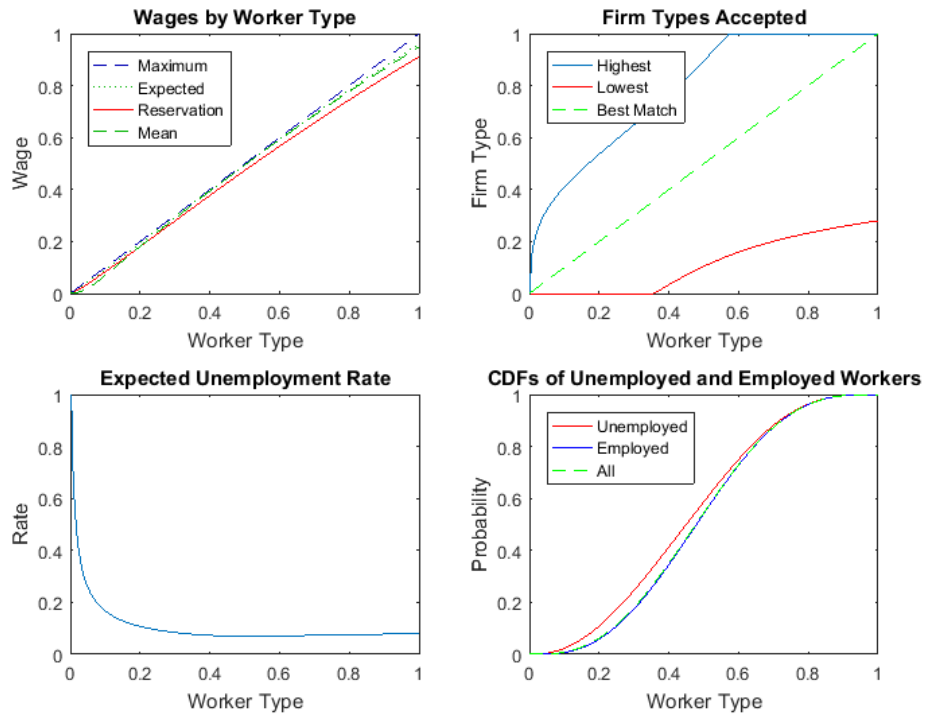


Figure 29: Decentralized equilibrium results for alternative parameterization (4).



firm types accepted. Increasing both parameters simultaneously leads to lower mismatch acceptance, but no change in the unemployment rates or duration of unemployment.

Table 9: Labor market moments obtained from data vs. those from the simulated models.

Moment	Data	(1)	(2)	(3)	(4)
$\bar{u}$	8.35%	8.44%	8.35%	8.35%	7.92%
$\mathcal{H}$	20.71%	20.71%	20.85%	16.50%	22.49%
$\mathcal{D}_{90,10}$	1.0331	1.0331	1.0331	1.0331	1.0351
$\mathcal{D}_{90,50}$	1.0158	1.0138	1.0124	1.0145	1.0181
unemp. dur.	4.8296	4.9270	4.8615	6.18	4.6108

## Skewed Skill Distributions

It is not likely that the distributions of all cognitive and non-cognitive skills are symmetric. For instance, one might imagine that the distribution of leadership ability has a long right tail, so that a small percentage of individuals are extremely skilled in this area. On the other had, it is possible that a skill such as English communication is skewed to the left, when the population of interest is the U.S. labor force. To illustrate how a skewed skill distribution affects the equilibrium results, I provide calibrated equilibrium results for two additional distributions,  $\beta(2, 5)$  and  $\beta(5, 2)$ . The three distributions used are compared in Figure 30. I re-calibrate  $\lambda$  and  $\delta$  through SMM, using the other parameters from the preferred calibration. Table 10 gives the calibrated parameter values.

Figure 30: Beta distributions for skill type.

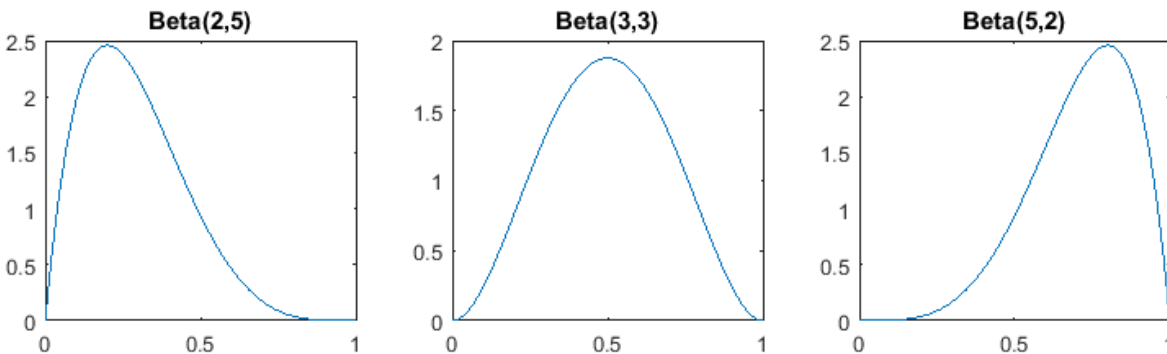
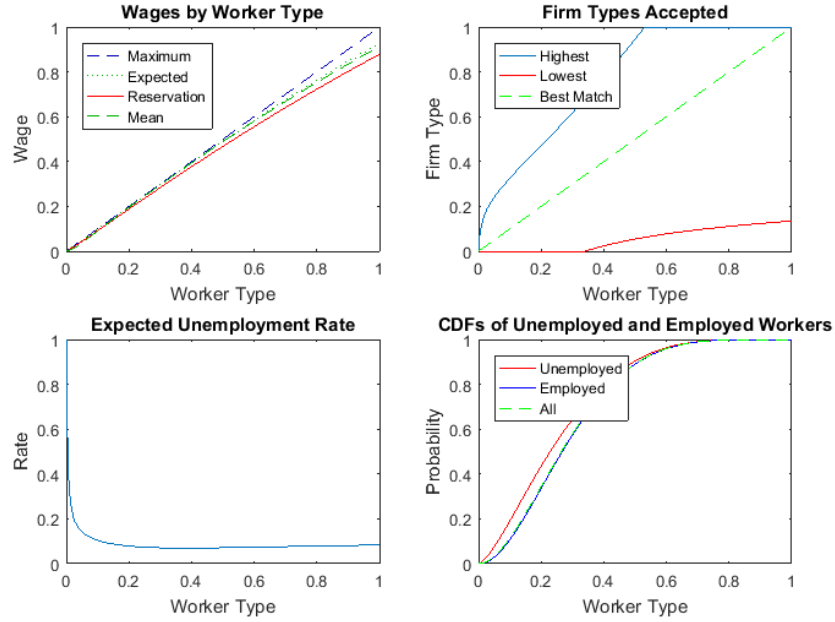




Table 10: Parameterizations for skewed distributions.

Parameter	Beta(2,5)	Beta(5,2)
$b_0$	0	0
$b_1$	0.4	0.4
$r$	0.001	0.001
$s$	0.0188	0.0188
$\lambda$	0.2546	0.2221
$\delta$	0.1625	0.1833
Targets	$\bar{u}, \mathcal{D}_{90,10}$	$\bar{u}, \mathcal{D}_{90,10}$

Figure 31: Decentralized equilibrium results with  $F(x) \sim \beta(2, 5)$ .



Choosing  $\lambda$  and  $\delta$  to match  $\bar{u}$  and  $\mathcal{D}_{90,10}$ , the model is able to generate values for  $\mathcal{H}$ ,  $\mathcal{D}_{90,50}$ , and unemployment duration that are comparable to those from the alternative parameterizations in the previous section.

Figure 32: Decentralized equilibrium results with  $F(x) \sim \beta(5, 2)$ .

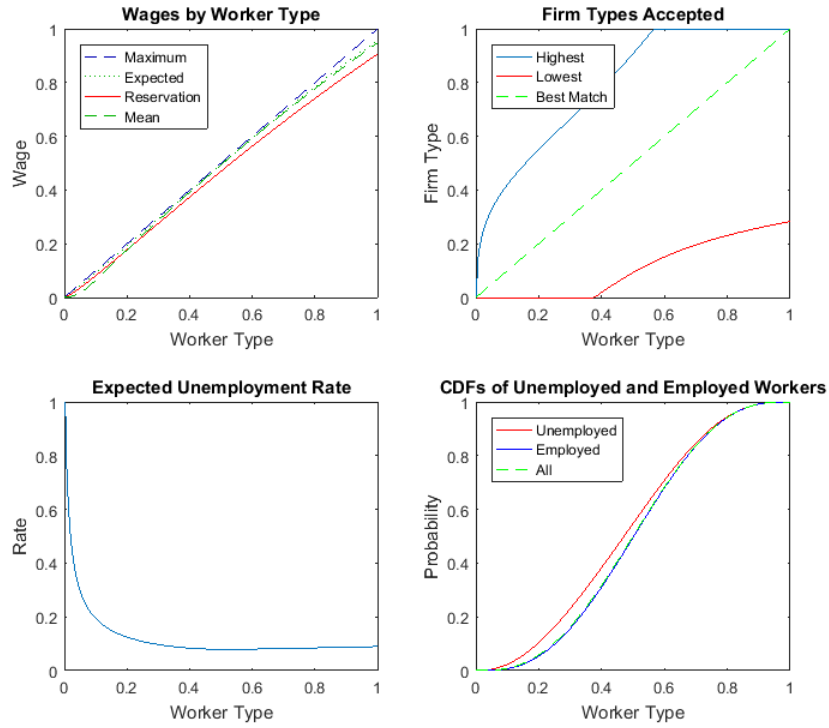


Table 11: Labor market moments obtained from data vs. those from the simulated models.

Moment	Data	Beta(2,5)	Beta(5,2)
$\bar{u}$	8.35%	8.35%	8.09%
$\mathcal{H}$	20.71%	21.69%	21.45%
$\mathcal{D}_{90,10}$	1.0331	1.0331	0.9658
$\mathcal{D}_{90,50}$	1.0158	1.0286	0.9837
unemp. dur.	4.8296	4.9216	4.6914